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Editor-in-Chief—William David Reeve, Teachers College, Columbia University.
Associate Editors—Vera Sanford, State Normal School, Oneonta, N.Y.
W. S. Schlauch, School of Commerce, New York University.

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THE MATHEMATICS TEACHER

Volume XL — Number 3

Edited by William David Reeve

Student Teaching in Mathematics

By W. D. Reeve, *Teachers College, Columbia University, New York 27, N. Y.*

and

Homer Howard, *Radford College of Virginia Polytechnic Institute, Radford, Va.*

I. Why Have Student Teaching[1] for Mathematics Teachers?

Any learning is an active process, a matter of personal development which comes about largely as the result of surmounting difficulties. Thus *student teaching* is a means of promoting growth through meeting issues, situations, and problems, under wise guidance.

A teacher in service has many issues[2] to face. His judgments concerning them are more certain if he has lived intensively in the realm of these issues. The more he becomes aware of them during his period of preparation, the stronger he is to cope with them when he assumes responsibility for a learning situation.

In a somewhat indirect way, he may have faced important issues in his previous courses in mathematics and education; in student teaching, either he actually sees experienced teachers making necessary choices concerning their teaching, or he makes them for himself as he proceeds with his own work.

No teacher can predict the situation, the problem which may arise in the schoolroom. Many teachers strive to keep their control so rigid, their technique so stereotyped, as to reduce the chances for the unexpected to happen, especially in connection with subject matter. However, events cannot be completely charted in advance. As for those teachers who encourage pupils to take part in directing their school affairs, they can only predict that whatever happens will have many elements of novelty, so that to meet the unexpected wisely and sanely often requires all the resources they have of prudence and good humor. Teachers of all types are likely to agree, however, that it becomes easier for them to meet challenging situations after they have experienced related difficulties and solved them. Hence the value of student teaching becomes apparent. It provides opportunities for beginners to try their hands at teaching in such a way that their failures may not be too costly either to them or to their pupils; so that their successes may help them to face new situations with confidence.

This kind of experience seems to be es-

[1] The term *student teacher* used throughout this discussion is somewhat at variance with the customary term in some institutions, of which Teachers College is one. It is used here to conform to the resolutions adopted by the Directors of Student Teaching at their Cleveland meeting in 1920.

[2] Later in this discussion, some issues of special concern to mathematics teachers are discussed.

pecially desirable for mathematics teachers. Mathematics, quite naturally, attracts to it mostly those who enjoy being logical and precise in their thinking and living. Such people often are prone to lose sight of what tumultuous, helter-skelter affairs growth and learning are. In viewing the beauty and symmetry of the final products of mathematical thought, inexperienced mathematics teachers forget the random, even disorderly, flow of inductive-deductive processes that brought them about. They are not likely to remember how unsystematic is childhood. Thus, they attempt to deal with immature minds as though they were trained, experienced. Young teachers almost invariably try to teach too much and too hard mathematics. In trying to teach special abilities, they seem to be unaware that simple material is better than complex, difficult material.[3] Such considerations as these make it especially desirable for beginning teachers to start their work under guidance from one who has learned to make these adjustments between his subject (mathematics) and his subjects (the pupils).[4]

That student teaching is considered of value to mathematics teachers and others is shown by the statutes and board regulations established in the various states of the United States. The state of New Jersey, for example, requires that candidates for teaching certificates shall have had one hundred and fifty clock hours of teaching under supervision.[5] The state of New York requires from two to six semester hours credit for student teaching, and specifies that twenty clock hours of actual teaching must be done for each semester hour of credit granted.

Furthermore, much emphasis is put on this kind of preparation by universities and colleges as shown by their extensive provisions for such work. Mead[6] estimates that a total of 459 universities, teachers colleges, and other institutions of comparable rank were offering student teaching as early as 1926.

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics[7] gives its support to student teaching in these words:

> The most important element in professional training is student practice teaching, carried out under the most competent supervision that can be procured. The Commission considers this work so important that it urges that even greater attention be paid to it in the future than in the past. Work in practice teaching should be preceded by a good course in methods; and in this course special stress should be put upon the topics that are most intimately connected with the ideas, the concepts, and the basic processes of mathematics. Some phases of the work of the methods teacher should be carefully correlated with instruction that students receive in mathematics.

The National Survey[8] shows that in 1930–31 all teachers colleges and about 85% of colleges and universities required students preparing to teach to have experience in student teaching. Even more convincing than this statement are the testimonies of teachers in service. Those who have had this experience give it high rank in comparison with other courses in education which they have taken.

[3] E. B. Sullivan, *Attitude in Relation to Learning*, Ph.D. Thesis, Stanford University, 1924.

[4] Student teaching should serve at least to prevent this kind of situation observed in a public high school: A recently certificated graduate of a liberal arts college, who had taken mathematics as his major college subject, was attempting to teach second-year high-school algebra, using the set of notes he had taken in college algebra. The material in his notes was being presented to the children with little or no adaptation.

[5] Of these one hundred and fifty clock hours required by New Jersey, ninety must be spent by the student teacher actually in charge of classes.

[6] A. R. Mead, *Supervised Student Teaching*, Johnson Publishing Company, 1930, p. 16.

[7] The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics on *The Place of Mathematics in Secondary Education*, Fifteenth Yearbook, National Council of Teachers of Mathematics, p. 191. Bureau of Publications, Teachers College, Columbia University, 1940.

[8] *Special Survey Studies*, National Survey of the Education of Teachers, Vol. V, p. 120. U. S. Superintendent of Documents, 1933.

In the study, "Major Issues in Teacher Education," issued by the American Council on Education, the following assertion is made.

> There is no more doubt as to the value of practice teaching in teacher education than there is as to the value of interneship, in medicine.[9]

Because student teaching seems in harmony with the postulates that we learn by doing, that we become strong by surmounting obstacles, that we reach high degrees of skill through practice under favorable circumstances, it is not strange that there is a trend in some institutions[10] to center the work of educating teachers around student teaching, and to cut requirements for lecture courses to a minimum or to eliminate them altogether. At Teachers College, however, the practice is not so extreme. Here facilities are provided for student teaching for those who desire it and who are deemed worthy of the privilege.[11] Thus it is recognized as one of many available means for furthering the education of teachers. Finally, the old adage, "Teachers are born and not made," seems to be in need of further study. Surely there is something in training.

II. Who Shall Undertake Student Teaching and Where?

The *Teachers College Bulletin*[12] states that student teaching is open to those "who have completed with distinction certain required courses in their field of major interest and who in character, personality and general promise meet the requirements deemed essential for effective teaching." Thus, it is evident that certain professional qualifications enter into the selection of student teachers in the field of mathematics. Student Observation[13] and Teaching in Mathematics (Education 267PT) carries three points of credit for either the winter or the spring session. It is subject to certain general regulations given for student teaching in the *Bulletin.* Outstanding among these are the requirements that the candidate pass a test in speech, give evidence of good health by submitting to a comprehensive medical examination, and measure up to the specific requirements of the respective departments. Furthermore, it is expected that the candidate be a holder of the bachelor's degree. He may thus include the work of student teaching among his other offerings for the master's degree, although it is expected that such a course will prolong the time required to earn the master's degree. In addition to the general requirements of the College, the Mathematics Department stipulates that the student teacher be a full-time major in mathematics.[14]

Not only at Teachers College, but generally, student teaching is regarded more as a privilege than as a right. Limitations in facilities as well as consideration for the welfare of pupils in cooperating schools make this necessary. Mead[15] says: "No college entrance methods in use today provide such a guarantee," namely, that student teaching will be provided for all. Speech defects, physical defects, unsatisfactory personal appearance, and above all indifferent scholarship tend to bar certain applicants. However, at a recent meeting of supervisors of student teaching[16] it seemed to be the consensus that the emphasis in selection should be on recruiting likely candidates. rather than on rejecting poor ones. It was recommended that those responsible for the preparation of teachers should induce students of outstanding promise to consider teaching as a career, to

[9] American Council on Education, "Major Issues in Teacher Education," *American Council on Education Studies*, Series I, Volume II, Number 4, February 1938, p. 15.

[10] See the 1940–41 *Bulletin* of the University of Florida, as an example of such programs.

[11] *Teachers College Bulletin*, "Announcements of Teachers College, Columbia University, Winter and Spring Sessions, 1946–47," Thirty-seventh Series, No. 1, p. 36.

[12] *Ibid.*, p. 33.

[13] *Ibid.*, p. 107.

[14] *Ibid.*, p. 107.

[15] A. R. Mead, *Supervised Student Teaching*, Johnson Publishing Company, 1930, p. 293.

[16] Vacation Conference, National Association of Supervisors of Student Teaching, August 19–29, 1939, Pineville, Kentucky.

apply for the privilege of student teaching. Such a line of action, in the main, was considered preferable to administrators' setting themselves up as guardians to ward off from student teaching those whose appearance and past performance seems unpromising. Experience shows that many who, at first, may have seemed unlikely to succeed have later revealed sincere purposes and unsuspected powers, although of course this would not be the general rule.

Certainly, both the prospective student teacher and his advisers need to concern themselves with his readiness for student teaching, especially in the field of mathematics. Concerning the student's general qualifications, Mead[17] lists the following items as deserving attention:

First, physical attainments and enjoyment. The home, school, and college should all contribute to these.
Second, personal qualities and traits cultivated by previous experience.
Third, unless the reader includes in the concept "personal qualities" the religious and moral phases of life, these are to be provided for as fully as possible by home, church, school, and college.
Fourth, there is a group of qualities and abilities better provided for in the existing curricula than the first three items. In this group are found:

1. mastery of the elements of subject matter which the student teacher must teach,
2. wide acquaintance with materials in the subject to be taught which extends beyond the range of the minimal elements to be mastered,
3. wise acquaintance with closely related fields of knowledge and human activity,
4. what is now called a general education, derived from contact with teachers and subjects in the fundamental aspects of human knowledge such as the vernacular language and literature, the basic natural sciences, the social studies, and physical, religious, and philosophical studies,
5. preparation to the extent of mastery of certain educational principles and conceptions derived from theory courses, observation, and participation.

Turner lays down certain principles in his study, *The Training of Mathematics Teachers*,[18] which pertain to the selection of the student teacher inasmuch as a great part of his pre-service education is undertaken prior to making application for student teaching. These principles are well worthy of consideration:

1. Prospective mathematics teachers should receive a thorough course of training in mathematics.
2. This training should be given in a university or an institution of equivalent rank by teachers who are themselves mathematicians of outstanding competence and who appreciate and understand the difficulties inherent in mathematics, whether it be regarded as a subject of learning or of teaching.
3. Mathematics teachers should study the important branches of pure mathematics, mechanics, the history of mathematics, applications of mathematics, as for example study of its logical foundations, and particularly the essential connection between the various branches of advanced mathematics and their counterpart at the more elementary stage.
4. Mathematics teachers should make a less intensive study of some subject, preferably one closely related to mathematics.
5. The teacher, during his period of active service, should strive to progress in his acquaintance with and mastery of many aspects of mathematical knowledge.
6. A period of professional training is a necessary part of preservice training of mathematics teachers.
7. The content of this course of professional training should be organized principally for the purpose of training teachers for teaching mathematics.
8. Mathematics teachers should be equipped to teach at least a second (and preferably an allied) subject, and they should therefore undertake a course of professional training in this second subject.
9. This period of professional preparation of mathematics teachers should include some courses in the theory and practice of education, and in psychology.

The manner of securing an appointment as student teacher varies widely from place to place. According to Mead[19] the general custom is to fill out a blank known as an

[17] A. R. Mead, *Supervised Student Teaching*, Johnson Publishing Company, 1930, p. 290.

[18] I. S. Turner, *The Training of Mathematics Teachers*, Fourteenth Yearbook, The National Council of Teachers of Mathematics. Bureau of Publications, Teachers College, Columbia University, New York, 1939, pp. 218–219.

[19] A. R. Mead, *Supervised Student Teaching*, Johnson Publishing Company, 1930, p. 293.

"application for student teaching." This is submitted to a coordinator or administrator. If his action is favorable, the applicant is placed in a student-teaching position. At Teachers College the prospective student first applies to the major adviser in mathematics for admission to student teaching. If the adviser approves, the student's application is taken to the Executive Officer in Charge of Student Teaching. Possibilities of placement in cooperating or campus schools are studied. Among other things to be considered in connection with the course in student teaching is the question of the student's total program. Students who have a full schedule of classes besides student teaching do not do well in this work. As a rule, the total program at Teachers College which includes this kind of work should not run over twelve points; in some cases, the maximum number of points should be less.

Teachers College offers two types of situations for beginning teachers under guidance:

1. Campus schools: As at other institutions, these schools have as their primary purposes the welfare of the children who attend them. However, they also recognize as a distinct function the training of beginning teachers through actual contact with the children. Other purposes, not germane to this discussion, are that they shall provide facilities for observation of teaching techniques, and for experimentation and research. The schools so maintained at Teachers College are: the Horace Mann-Lincoln School, and the Horace Mann School for Boys.

2. Cooperating public and private schools: In the vicinity of New York City, as in many other sections of the country, facilities are provided for students to teach in certain public and private schools, in what might be termed laboratory situations. These cooperating schools provide not only the children to be taught but they also provide experienced teachers under whom the work is done. The teacher-training institution makes arrangements and designates individuals on its own staff who visit the students in their teaching situations. These visitors assist the beginners to interpret their teaching experiences, through conferences and seminars.

In either type of situation, whether in the campus or cooperating school, there is someone to whom the student teacher looks for guidance. His relations with this person, variously designated as his supervising, training, or critic teacher,[20] will largely determine the success or failure of the venture. First, the student observes this experienced teacher at work. Later, these roles are reversed. At all times, he seeks to share in the work of his "guide, philosopher and friend." There are papers to read; perplexed and discouraged individual pupils to coach; arrangements for school visits and visitors to be made; displays to be set up; extra-class activities to attend to; and a hundred and one things in which the student teacher can share. A more complete discussion of what he is expected to do in the situation to which he is assigned will be given later in this discussion.

At this point, attention is directed to some of the terms used in student teaching. Ordinarily, the student is called a *student teacher* or *practice teacher*. The experienced teacher in charge of the classroom, if in a campus school (frequently called a laboratory, demonstration, or model school is usually designated as a *critic* or *supervising instructor*. In cooperating schools not administered by the institution under whose auspices the teaching is to be done, this person is sometimes called a *training teacher*. In addition, there is a third person, usually called the *coordinator* or *director of student teaching*,[21] who makes the assignments and controls the entire plan for giving students opportunities to

[20] In campus schools the term frequently used is *critic teacher;* in cooperating schools, *training teacher;* although there is apparently no standard practice.

[21] At Teachers College this person is called the Executive Officer in Charge of Student Teaching.

see and take part in teaching at first hand. A fourth is the head of the mathematics department, who acts as an *adviser* in matters concerning the student teacher's previous training and the mathematical subject matter he will teach. He in turn goes, or delegates someone in the department who goes, to the cooperating or campus school to observe the student teacher at work, to confer with him and his supervising (critic) teacher. At Teachers College a weekly seminar is sometimes conducted for student teachers, at which their common problems are discussed. In other institutions the person who visits student teachers and conducts their seminar is frequently the same person as the adviser, the head of the mathematics department.

Now that teaching under guidance has become an established practice, certain procedures and relationships have been crystallized. Thus certain meanings have been attached to the terms *practice teacher*, *student teacher*, *critic*, *supervising instructor*, and *supervisor*, which are open to question. They imply relationships which are unacceptable, especially when they embody hard-and-fast notions regarding teaching relationships, teaching goals, techniques, materials, and bases for evaluation. In order to clear the way for new meanings, and relationships, we propose that the terms themselves be studied. It may be that simpler words would serve better: *Beginning teacher* might be used to designate the one who is undertaking teaching under special arrangement, usually for the first time. Because he is a student of teaching in a special sense it may be appropriate to call him a *student teacher*. Either is preferable to *practice teacher*, a term which has acquired some undesirable meanings. None of these designations is suitable for use by the pupils in the schoolroom. They should refer to him as a *mathematics teacher* or *instructor*. That he does not hold a teacher's license nor a permanent position as teacher should not deprive him of that responsible status. Similarly, the teacher who guides and assists him from day to day might be called the *guiding teacher*. These terms are suggested in order to imply a "two-way process" between the novice and the veteran, a relationship in which respect is mutual and experiences are shared.[22] Teachers in campus schools sometimes object to being designated as *critic teachers*, both because it seems to give them a position inferior to the members of the college staff and because *critic* over-emphasizes one aspect of their relationship with students. Thus they are likely to welcome some such change as this. For the mathematics supervisor representing the college, if any designation is needed, the term *college visitor* or *counsellor* might be used.

When the beginner is referred to as a teacher, or even as a student teacher,[23] the implication is that he has something to contribute, that he is not altogether the novice. This is truly the case. From the standpoint of experience, he has been in school himself from fifteen to eighteen years. He has come to know many teachers and teaching situations. Further, he has studied the principles underlying teaching, to some extent at least. Ordinarily he knows something of educational psychology. In addition, he has been judged a proper candidate for teaching; one who has reached the point of having a good working knowledge of the mathematics commonly taught in secondary schools and colleges; one who is likely to succeed in his dealings with children or adolescents. He has begun to

[22] Experienced teachers who have worked with student teachers are quick to testify that they have learned many valuable things from their student teachers, that they have encouraged mutual sharing of ideas because of the benefits that come to them as a result of such sharing. This is not an unusual fact since some novices may be actually or potentially superior to their elders in certain respects.

[23] The following designations will be used in the later sections of this discussion; as representative of current practice: *student teacher* for the one who is preparing to teach; *supervising teacher* for the person in either campus or cooperating school who guides the student teacher; *adviser* or *supervisor* for the representative of the institution under whose auspices the student teaching is done.

be aware of some of the issues involved in mathematics teaching.

III. What Issues Confront Teachers of Mathematics?

As soon as the student teacher has been placed, he begins to take part in the work of the mathematics classroom. At the start, it is true, this participation is somewhat passive. Nevertheless, he begins to be aware of certain issues, or cleavages in the teaching of mathematics. Concerning some of these he must make his own choices; concerning others he must conform to the choices of others, at least for the time being. These issues have to do with: the teacher's own relations with mathematics; his relations with pupils, their parents, and others directly connected with the school; and their mutual relationship with mathematics. These issues, in turn, lead to problems related to his and the children's relations to society.

Important as they are, this discussion does not attempt to deal specifically with those issues having to do with society. Rather, it seeks to sharpen the issues having to do with the teacher's relations with children, their parents and others directly connected with the school, and with mathematics. Some of these are:

1. Shall we consider mathematics as a necessary end in itself for all the children; or, rather, as one of many means to promote larger ends?
2. Shall we emphasize the purely logical, deductive side of mathematics more than the physical, experimental side; or vice versa?
3. Shall we stress manipulation or reasoning?
4. Shall we strive to keep the traditional branches of mathematics intact, or shall we teach in terms of such notions as *dependence, variation, probability,* drawing at need from arithmetic, algebra, geometry, trigonometry, and the calculus?
5. Shall we seek to preserve mathematics as distinct from other school subjects or shall we correlate it with such fields as the social studies or natural sciences?
6. Shall we rest on our faith that mathematics, learned for itself, will find uses in broad ranges of life activities; or must we plan from day to day to manifest the applications of mathematics, expecting no transfer otherwise?
7. Shall we teach in terms of lessons from day to day, or shall we teach in terms of larger and more varied experience?
8. Shall we develop mathematics step by step, in logical sequence, sometimes in advance of pupils' realization of need; or shall we postpone our teaching efforts until pupils have encountered difficulties which call for new mathematical insights?
9. Shall we hold our pupils to certain pre-determined standards, grade-norms; or shall we encourage them to select individual goals toward which to progress at their own rate?
10. Shall we work within the current curricula, responding when necessary to pressures that arise from time to time (such as those that promoted the teaching of business arithmetic and thrift); or shall we look for new curricula, made from well-stated objectives, fashioned in terms of so-called pupil needs?
11. Shall we be more concerned with the beauty and symmetry of mathematics itself, or shall we give thought principally to whatever human values we may find in the teaching-learning situation?

The beginning teacher, certainly, cannot and is not expected to be aware of all these issues when he first enters student teaching, although he doubtless knows about some of them. At first, he is caught up in the momentum of the school as he finds it. By right, his initial acquaintance with children is very informal. If seats are in rows and work proceeds by recitation and assignment, he sits in the rear of the room, observes the interplay between class and teacher. He learns to associate the names on his data sheet with the young persons before him. Later he reads pupils' papers. Gradually he relieves the regular teacher (except in rare cases) of much of the routine work. Or, if the group is differently organized, along somewhat more flexible lines, then he may be taken in hand by the pupils themselves. They may explain to him what is being done, and how. They will, perhaps, introduce him to each member of the group; they will show him around the building and acquaint him with the arrangements for living together in the school. He thus becomes one who is assisting, sharing, growing with them. As he becomes known to both pupils and their teachers, his participation increases. Soon the experienced teacher releases responsi-

bilities to him until, finally, he is entirely charged with them. Thus, in the latter case we have the three phases of the student's beginning teaching—namely, observation, participation, and student teaching[24] —merged into a continuous process. In the former type of situation, these phases may be marked off into fairly sharp divisions: so many days in which to watch the activities of the group, taking careful notes the while; so many days in which to assist the regular teacher by attending to details of lighting, heating, ventilating, by reading pupils' papers, checking attendance, giving special encouragement to laggards and those who have been absent. All the while he continues to observe the classroom work of the regular teacher. Finally, so many days are designated in which to present and teach the materials he has chosen and prepared under the guidance of the one who is his critic, in fact as in name.

Whatever the set-up, it will be as he finds it and not as he makes it. Thus it might well be asked, what has he to do with issues? The answer is this: If his experiences are to be fruitful, he must interpret them. He must relate them to the values he has acquired and is acquiring. Even though his own range of choice may be limited, at least he will see choices being made. He must become aware of the whole scheme of values underlying the practices of the mathematics classroom, both explicit and implicit. This he will do through reflection, conferences, inquiries, and many other means of reconstructing his day-to-day experiences. He will begin to see whether the children are, in the main, *ends* in themselves or *means* to the attainment of certain ends, such as high performance on extra-mural tests; whether they are being led to *discover* relationships and to seek comparisons as well as being required to use definitions, axioms, postulates in formal mathematical statements; whether their attention is centered on developing a high degree of skill in computation, somewhat to the neglect of practice in reasoning; whether such notions as *dependence, induction, correspondence, probability,* and *variation* form the basis for the teacher's attack rather than the logically arranged topics of arithmetic, algebra, and the other branches; whether marks and grades, rewards and punishments, praise and blame are used to stimulate greater study of mathematics; whether the work is organized by daily lessons or in comprehensive units not necessarily related to time spent. It may seem to him, perhaps, that there is no great amount of conscious choosing, either by the children or by the teachers, in any of the work being done in mathematics. He may then conclude privately—for the time being and until he has reason to think otherwise—that whatever values are present are there by tradition rather than by free choice. Even then there is choice. For example, he must decide whether he will teach as he sees children being taught, and thus continue the tradition, or whether he will try to find new emphases and new values. In this way, his student teaching becomes a time of real study and not merely an exercise in recipe-gathering. He is concerned with the issues that confront all mathematics teachers.

From such study of issues in the teaching of mathematics it is but a short step to much larger considerations, such as: What are my duties to society as a teacher and as a person? What are the proper functions of a school in society? Must the school always follow or should it lead, at least some of the time? What does it mean to live in a democracy? What is the role of intelligence in modern society? What is the contribution of mathematics to intellectual affairs?

These and many more questions concern the teacher, whether beginner or veteran. To the extent that he becomes aware of them, it may be said, to that extent he is growing.

[24] A. R. Mead, Supervised Student Teaching, Johnson Publishing Company, 1930, p. 164.

IV. What Mathematics Is of Most Worth?

When the first days of his observation have passed, the student teacher begins to plan for the time when he will be granted the privilege of conducting class work over a period of time. By now he is acquainted with the children. He knows their names, something of their tastes and interests, and quite a lot about their achievements and their attitudes toward mathematics. Under guidance from his supervising teacher, he starts planning for actual teaching.

In some situations he is given considerable choice as to what he will teach; within the limits imposed by the capacities and attainments of his pupils, he is free to select teaching materials. In other situations he has little or no choice; the matter is settled by the training or supervising teacher's telling him what he will teach. It may be, of course, that the latter has very little choice himself in the matter because of the influence of extramural examination boards. In either case, however, he is faced by immediate questions concerning the curriculum, whether in connection with making more or less free choices of procedures and subject matter, or in connection with interpreting the choices that are being made for him.

Logically the first question is: What is "curriculum?" This term, like many others used by educators, is found in a wide range of meanings and in varying contexts.[25] Narrowly used, it means a course of study, a sequence of topics to be learned or a series of units to be mastered. Broadly used, it embraces "all the experiences children have under guidance of teachers."[26]

With the narrower concept of curriculum in mind, the student becomes concerned with the lessons likely to come in the series about the time his student-teaching experience occurs. Thus he, or his supervising teacher for him, will consult the textbook in use, the course of study, or the syllabus, or all of these, to guide in making plans. When the broader concept is kept in mind, they discuss the interests, tastes, needs, activities, attitudes, previous and present experiences of the children with a view to guiding them into mathematical experiences such as are likely to be of the most worth to them.

Thus the approach may be from one or several angles. Bruner[27] recognizes the following:

1. Child experience approach
2. Creative values approach
3. Frontier thinkers approach
4. Socio-economic approach
5. Social values approach
6. Social statistics approach
7. Educational shortages approach
8. Emotionalized attitudes approach
9. Adult needs approach
10. Activity analysis approach
11. Objectives approach
12. Scientific approach
13. Present practice approach.

A glance at this list may reveal that the approach is simply the way we attack the problem. It is determined by the values that we hold. For example, the "child experience approach" would be to make plans based on the current life of the children, the experiences they are having. In such an approach it might be said that the teacher would see what mathematics could be "got out" of what is going on—games

[25] Such variations in meaning are particularly irritating to those who love mathematics. In their favorite study they are accustomed to terminology which, if not absolutely exact, is limited in meaning to a high and satisfying degree. On the other hand, in educational, philosophical writings they constantly meet words which convey a variety of meanings. Worse, they even encounter authors who use the same word with different meanings, all within the same context! In the interest of tolerance and growth of understanding, it may be said that perhaps some of these vagaries in the meanings of words are necessary to growing concepts, in education or any other field.

[26] Hollis L. Caswell and Doak S. Campbell, *Curriculum Development*, American Book Company, 1935, p. 69.

[27] Herbert B. Bruner and C. Maurice Wieting, "A Tentative List of Approaches to Curriculum and Course of Study Construction with Suggested Bibliographies." Mimeographed brochure, Curriculum Laboratory, Teachers College, Columbia University, 1939.

played, parties planned, allowances spent and accounted for, distances walked, time spent, and the like—rather than what mathematics could be "put into" their experiences. Such an approach would be somewhat unusual except in connection with the teaching of arithmetic in an elementary-school activities program.

Closely related to this approach is the "creative values approach," in which emphasis is put on all sorts of creative activities. Such approaches are not uncommon with some mathematics teachers. They seek to have children engage in finding, making, discovering. Thus a formula might be *created* by a learner, as far as he is concerned, even though it had been in use for many centuries, unknown to him. However, the emphasis in such an approach is more likely to be on making models, drafting devices, alidades, telescopes, transits, slide rules, and the like, rather than on developing formulas and other generalizations.

In some instances there might be students and supervising instructors who would employ the "frontier thinkers approach." Here the initiative for doing certain things would come from a group of thinkers who are skeptical of all values, who tend to weigh and sift all school practices, no matter how time-honored. These thinkers prefer the way of reason and experimental techniques to the way of authority and traditional procedure. Mathematics has its own group of frontier thinkers.[28] Reflection shows this approach to be mutually inclusive with the "child experience" and "creative values" approaches, since the frontier thinkers favor those experiences which meet the needs of children and which foster their growth through creative activity. However, to give some notion of this approach, it is necessary to say more than this. Above all, the frontier thinkers emphasize the worth of the individual and his responsibility to society. Hence a curriculum based on their conceptions would emphasize that mathematics which tends to promote a better social order. Such mathematics very likely would include phases of statistics, economics, and logic.[29]

At the present time, especially in the larger systems of public schools, the student teacher is likely to find that the "present practices," the "objectives," and the "scientific" approaches are employed in the development of the mathematics curriculum. In the first of these approaches, the teacher studies the textbooks, the courses of study, the techniques in use in outstanding school systems, the recent reports of important local or national committees; then he organizes content, sequence of topics, and classroom practice according to these models. The "objectives" approach is a somewhat related way of attack. It too is a highly systematic way to proceed. The teacher, or other curriculum maker, sets up a list of desirable objectives, often in considerable detail. In doing so, he draws on his knowledge of children, of mathematics, and of society.[30] In many instances he makes use of lists of objectives already available. Once he has identified and listed the objectives, he selects the materials he believes will aid his pupils in attaining them; next, he decides upon the teaching-learning techniques he will use; finally, he constructs testing devices by which to determine whether the children have reached the goals formulated at the outset.[31] The

[28] The work of Harold P. Fawcett might be cited as an example of this approach. See *The Nature of Proof.* Thirteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, 1938.

[29] See Nathan Lazar, "The Importance of Certain Concepts and Laws of Logic For the Study and Teaching of Geometry." THE MATHEMATICS TEACHER, Vol. XXI (March, April and May) Nos. 3, 4, and 5, pp. 99–113, 156–174, and 216–240.

[30] The "objectives" approach is closely related to the "socio-economic," "shortages," and "adult needs" approaches.

[31] See Section V for discussion of techniques and Section VI for comments on evaluation.

"scientific" approach is closely akin to the two just mentioned. It emphasizes the use of standard test results. It proceeds to build the curriculum in terms of the achievement of a large number of children of given ages and grades, as expressed in norms and standards derived from test data obtained from large groups of children. In a sense, the aim of the scientific approach is to engender the ability to do the things that standard tests have shown that the so-called average student can do at a given age or grade level.

Of the three approaches, the "objectives" approach seems the most favored in the literature on the teaching of mathematics. It provides the logical mind with a satisfying sequence concerning the teaching learning process. This sequence may be expressed by the following diagram:

LEARNING MATERIALS (EXERCISES) → LEARNING ACTIVITIES → LEARNING OUTCOMES (OBJECTIVES)

The assumption here is two-fold:

1. If the learning materials or exercises set by the teacher are well motivated, they will lead to learning activity on the part of the children.

2. If the learning activities they engage in are appropriate to the objectives, then the children will reach the desired learning outcomes, or objectives.

In giving attention to the "objectives" approach to curriculum development, the student teacher will be interested in lists of objectives made by others, many of them the results of very painstaking techniques. For a discussion and list of objectives in mathematics he would do well to consult Smith and Reeve,[32] the Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics,[33] Schorling,[34] and many others who have formulated mathematics objectives. However, there are some dangers in carrying this approach to extremes.[35]

An example of material which is most valuable for setting up mathematics objectives is the brochure, *Mathematics Required for Science Work*, prepared by Lauwerys.[36]

As he becomes conscious of his need for a wider range of materials, the student teacher discovers the curriculum laboratory, especially if he is working in one of the larger school systems. Here curricula of all sorts are studied. These, together with texts, bibliographies, and other materials, are kept on file. At Teachers College, the Curriculum Construction Laboratory[37] has on hand more than 40,000 courses of study, available for inspection

[32] David Eugene Smith and William David Reeve, *The Teaching of Junior High School Mathematics*, Ginn and Company, 1927, pp. 22–90.

[33] Joint Commission of the Mathematica Association of America and the National Council of Teachers of Mathematics, *The Place of Mathematics in Secondary Education*. Fifteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, 1940.

[34] Raleigh Schorling, *A Tentative List of Objectives in the Teaching of Junior High School Mathematics with Investigations for the Determining of their Validity*, George Wahr, Publisher, Ann Arbor, Michigan, 1925.

[35] *Mathematics in General Education*—A Report of the Committee on The Function of Mathematics in General Education for the Commission on Secondary School Curriculum of the Progressive Education Association. D. Appleton-Century Co., 1940, p. 342.

[36] J. A. Lauwerys, *Mathematics Required for Science Work*, unpublished mimeographed brochure, Mathematics Department, Teachers College, Columbia University, 7 pp.

[37] The facilities of the Curriculum Construction Laboratory are not only available to student teachers who are connected with Teachers College, but also to others. Certain of its materials, such as lists of courses of study, may be procured from the Laboratory by mail.

In addition, its workers have developed a technique for judging them. Thus many courses of study in mathematics may be studied in the light of the ratings given them by workers in this field.

Whatever approach the student and his supervising teacher decide to use, they are duty-bound to have in mind not only the curriculum but the learners themselves, their physical environment, and the society in which they are living and will live. Probably no one approach is enough to insure that these considerations will be kept uppermost. Hence it is recommended that an eclectic approach be used, one which consciously chooses the best in the teaching of mathematics.[38] Such an eclecticism will arise out of those enduring frames of reference listed by the report of the Joint Commission[39] already referred to, somewhat as follows:

1. Human needs for food, clothing and shelter; for security from anxiety, disease, and drudgery.
2. Human desires to create something: architecture, pottery, textiles, pictorial and sculptured, objects of art and of ordinary use; poetry, music, fiction.
3. Human needs for idealism, worship, a spiritual life.

The mathematics curriculum may be viewed in another way: If a child is living, and likely to continue to live, in a society where obedience, conformity, respect for constituted authority are the most useful social attitudes, then certain emphases in the teaching of mathematics—certain content, certain techniques of learning and certain ways of giving evidence of learning—will be appropriate. On the other hand, if a child is living in a society where creativeness, individual initiative, and personal responsibility for the good of all are most needed traits, then it is likely that certain other emphases will be more desirable. In either case, teachers must see to it that the mathematics they are teaching, as much as lies within their power to choose, be such as will make the natural world become real to children in ever-widening reaches, that it will help them understand their own natures better, will cause them to take more responsibility as members of society. Certainly the mathematics presented should be free from "dead-wood," obsolete materials or materials that are too difficult for the learners at a given stage. At every point, the student must maintain a healthy skepticism concerning the mathematics he sees being taught and that which he attempts to teach, whether he has any choice in the matter, or not. He must concern himself with the question: What mathematics is of most worth?

Hence the student teacher asks: "What mathematical topics are obsolete?" Here again rise some of the issues discussed in Section III. If mathematics is viewed as that which strengthens and disciplines the mind, if it is held that the values achieved in studying mathematics are transferred directly to other fields of action, even though such fields are apparently unrelated to mathematics, then the topic studied matters little; only, the harder the better. Some schoolmasters would go so far as to say, the more distasteful the better. Nevertheless, we would find general agreement that extracting cube roots by the once-familiar arithmetical process is now obsolete. As a process, it takes a lot of time, its applications for the layman are few, and its results may be obtained far more quickly by other means. Other topics are mentioned in this connection by Smith and Reeve,[40] such as: elaborate exercises involving factoring in algebra, finding the highest common factors of lengthy polynomials, complex fractions, and various tricks that lead to little[41] that is worth much.

[38] Also see the recommendation of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, op. cit., pp. 10–24.

[39] *Ibid.*, pp. 19–20.

[40] David Eugene Smith and William David Reeve, *The Teaching of Junior High School Mathematics*, Ginn and Company, 1927, pp. 181–187.

[41] For a discussion of how certain types of factoring and other relatively useless materials crept into the mathematics curriculum, see Hobart Franklin Heller, *The Evolution of Factoring*. Keystone Publishing Company, 1940.

The question of what mathematics to select brings the student teacher and his advisers to a consideration of mathematical aims. Evidently, mathematical experience is relative. It is good if it promotes worthy aims and objectives; otherwise not. But what about such considerations for the beginning student? Even seasoned teachers are confused by the volume and diversity of the literature on mathematical aims.[42] To the novice probably the worst feature in this situation is the apparent impossibility of teaching in terms of broad, general aims as, for example, the utilitarian, disciplinarian, or cultural aims set up by the National Committee on Mathematical Requirements (1923).[43] In other words, one cannot hit the bull's eye unless he aims at it.

The key to this problem seems to be this: Of course, we cannot teach directly in terms of such aims. They are ultimate goals. However, we *can* teach in terms of immediate objectives. Every day the good teacher identifies and formulates specific, immediate objectives for the work at hand. These he holds or discards as they seem to measure up to the criteria: Are these objectives related to the broader aims toward which we are working?

Very broadly conceived, teaching-learning aims might be those in vogue a few years ago:

1. Health
2. Command of the fundamental processes
3. Worthy home membership
4. Vocation
5. Citizenship
6. Worthy use of leisure and
7. Ethical character[44]

Still general, but more focused on mathematics, are such aims as the following abilities:[45]

1. To discover certain rules and to translate these into formulas.
2. To translate formulas into rules.
3. To evaluate formulas.
4. To derive one or more formulas from a given formula.
5. To represent by a graph any simple formula.
6. To understand the dependence of one quantity upon another.
7. To work with ordinary, simple formulas.

We might take, for example, the student teacher who is planning to teach a group of youngsters the use of logarithms. Let us say that he intends to use the "objectives" approach discussed in this chapter, that he is mindful of the great ultimate objectives laid down by the National Committee on Mathematical Requirements.[46] Accordingly, he might put on paper the following specific objectives:[47]

Special abilities:

1. To pronounce, spell, and use in appropriate context such words and terms as: *characteristic, mantissa, logarithm, reciprocal, anti-logarithm, base, power, exponent, root, exponential-form equation, logarithmic-form equation, interpolation, mantissa table*, and the like.
2. To distinguish the characteristic and the mantissa of a given logarithm.
3. To find the characteristic of the logarithm of any number greater than 1 by counting the integral places in the number.
4. To find the characteristic of the logarithm of any number less than 1 by counting the number of zeros immediately to the right of the decimal point.
5. To find the logarithm of 1.
6. To find the appropriate mantissa for a given number by consulting the mantissa table.
7. To find the anti-logarithm of a given mantissa by consulting the mantissa table.
8. To interpolate when finding mantissas

[42] In this connection see William Betz, "The Confusion of Objectives in Secondary Mathematics," THE MATHEMATICS TEACHER, Vol. XVI (December 1923) No. 8, pp. 449–469.

[43] National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education*, The Mathematical Association of America, 1923, p. 5.

[44] National Education Association, Commission on the Reorganization of Secondary Education, *Cardinal Principles of Secondary Education*, Government Printing Office, 1918, pp. 51–60.

[45] David Eugene Smith and William David Reeve, *The Teaching of Junior High School Mathematics*, pp. 67–68, Ginn and Company, 1927.

[46] National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education*. The Mathematical Association of America, 1923.

[47] The general categories used here are those proposed by Henry C. Morrison in *The Practice of Teaching in the Secondary School*, University of Chicago Press, 1931.

of numbers and when finding anti-logarithms of mantissas.

9. To add logarithms when multiplying anti-logarithms; to subtract logarithms when dividing anti-logarithms.
10. To divide logarithms by the appropriate number when extracting roots; to multiply when raising to powers.
11. To make use of -10 or some other negative quantity in finding the characteristics of logarithms of numbers less than 1.
12. To locate the decimal point in anti-logarithms by considering the characteristics.
13. To find the anti-logarithms of logarithms from whose characteristics 10, or multiples of 10, or other quantities are to be subtracted.

Understandings of:

1. The relation of a system of logarithms to its base (especially the relation of the commonly taught system to 10).
2. The relation of the exponential form of expressing power equations to the logarithmic form of expressing such equations.
3. The meaning of *exponent* as related to the meaning of *power*.
4. The relation of multiplication, division, raising to powers, and extracting roots by means of logarithms to the laws of exponents.
5. How mantissa tables are constructed.
6. The processes involved in multiplying, dividing, raising to powers and extracting roots by means of logarithms.
7. Why characteristics change and mantissas remain the same when anti-logarithms are multiplied, or divided, by 10.
8. How logarithms of numbers less than 1 may be written in such form as to make use of -10.
9. The relation of the slide rule to logarithms.
10. The processes involved in making tabular interpolations.
11. The advantage of using a stereotyped form of equation when working with logarithms.

Appreciations for:

1. The work of Napier and Briggs, each of whom contributed much to the invention of logarithms.
2. The power of mathematics in that it includes such economical means of getting work done as logarithms.
3. The part played by logarithms and related mathematical processes and devices in developing such sciences as navigation, astronomy, engineering, and economics.
4. Algebra as a branch of mathematics which furnishes the essential framework for the study of logarithms.

Having made sure that these specific objectives, or others like them, conform to the criterion that they be related to the general aims toward which their work is pointed, the student and his supervising teacher proceed to plan experiences which will be helpful in reaching them.

In contrast to the systematic procedure just illustrated, the student teacher may take a somewhat different approach. He may say to himself: I am trying to teach a way of living. This way includes being prompt and reliable. It includes such traits as: being cautious in what I say and how I say it; being honest and dependable, able to budget my time and energies so as to meet my obligations; being fair and decent about human concerns; thinking rigorously; weighing and considering results whenever possible before taking action; making good estimates when necessary; learning to be calm in the face of difficulties. These and other admirable traits are likely to be exhibited by those who really know and like mathematics. What situations, what experiences shall I promote that my pupils may be led to acquire such traits?

With such thoughts in mind, he will attempt to guide the children into work with mathematics. He will be mindful of past experiences as well as present interests, both theirs and his own, all in terms of the necessary sequences of the mathematics they are to study. He will attempt to use mathematics to make them sensitive to their environment; to give them opportunities to exercise choice, to make judgments, decisions; to encourage them to accept responsibility for their choices. In so doing, he may lead his pupils into such situations as those described by Fawcett[48] in writing of his work with pupils in geometry. He probably will organize his teaching in an attempt to refine their thinking in problem situations. He may use such

[48] Harold P. Fawcett, *The Nature of Proof*, Thirteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, 1938.

unifying concepts as the following, proposed by the Mathematics Committee of the Progressive Education Association Commission on the Secondary School Curriculum:[49]

1. Formulation and solution of problems.
2. Gathering and organizing data.
3. Approximation.
4. Function.
5. Mathematical operations.
6. Proof and generalization.
7. Symbolism.

He will encourage his pupils to attack problems of importance to them; to analyze their problems, to formulate possible solutions; to gather data; to interpret their findings and apply them to useful purposes. In the course of such experiences, needs will arise for better, faster means of computation. Logarithms and the slide rule may provide such means. Thus the children are led to "discover" them, to make them their own. Some will not be content to acquire the ability to compute by means of logarithms, but will want to go on to study the mathematical basis of logarithms.

For such children, the expansion of the intellectual horizon is a real need. Curiosity, ambition, love of better understandings drive them on, set up tensions which the teacher must help them to release. For example, the logic involved in geometry is not enough; they want to go on to actual concrete experimentation with space. It will not satisfy such learners to read the reports of a Gallup Poll concerning a presidential election; they will want to find out what assumptions, what processes lie back of such reports.

Naturally, the student teacher who could provide for such a variety of interests and needs would be rare indeed. Even so, he can at least strive to maintain attitudes *favorable* to new and wider interests on the part of pupils. He can take stock not only of himself but of the community, to discover new resources to meet new developments. In this the supervising teacher is his main dependence, both in advising him concerning what leads to follow and how to follow them.

In such mathematics teaching, tool operations are elementary forms of mathematical thinking. They are to be mastered in order to promote, as Mursell says, "organizing of mathematical intelligence."[50] To say this is not to say that the facts of mathematics are not of themselves the essential subject-matter of mathematics teaching. They are the essentials. However, these facts and the way they are related to each other are to be *experienced* in as rich a way as possible. As the student teacher grows, he becomes more and more alive to new ways of interpreting and applying—experiencing—mathematical facts.

Some of the things the student teacher might plan in keeping with the broader aims of mathematics are:

1. Studying probability in terms of current superstitions connected with the number 13 and Friday; also, in terms of movie "bank nights," bingo, bridge hands, and other familiar situations involving chance.
2. Studying mechanics, both deductively and experimentally, so as to make such structures as the George Washington Bridge intelligible.
3. Studying indirect measurements and how they are used in surveying and navigation.
4. Making surveys, then organizing the data obtained in terms of the most representative average (mean, median, mode) and measure of dispersion (standard deviation, quartile deviation, range).
5. Computing the true rates of interest charged by loan "sharks," furniture dealers, and others, on deferred payments.
6. Gathering and organizing data from certain types of psychological experimentation.
7. Learning to "spot" distances.
8. Making studies of home and community budgets.
9. Studying life and fire insurance premiums and the statistics on which they are based.
10. Studying the financial end of cooperative stores and credit unions.

[49] Progressive Education Association, Commission on the Secondary School Curriculum, Committee on the Function of Mathematics in General Education, *Mathematics in General Education*, p. 59, D. Appleton-Century Company, 1940.

[50] Reprinted from *The Psychology of Secondary School Teaching* by James L. Mursell, by permission of W. W. Norton and Company, Inc. Copyright, 1932, by the publishers.

11. Using and interpreting formulas in the physical sciences.
12. Studying biological data to determine the number of permutations which are possible when certain reproductive organisms unite.
13. Finding maximum volumes obtainable in the various regular solids when surface areas are constant. (Such investigations could range all the way from food containers to dwelling houses.)
14. Computing the relative values of foods in terms of various ratios: cost to vitamin content, cost to calorie value, cost to protein content, and so on.
15. Studying vocational and productive trends.
16. Finding and interpreting data concerning the use and consumption of natural resources.

Some students and supervising teachers would hesitate to reach out into such varied activities as those just listed. Many teachers feel that the time of mathematics pupils should be given to the unique processes of mathematics, that to undertake investigations in other fields, while stimulating and educational in a general sense, would break into pupils' mastery of those orderly sequences which are necessary to learning mathematics. Certainly, such a course would mean that the teacher would need to leave out or gloss over some materials which ordinarily take up a great portion of the class time. Such a course would also presuppose a teacher well trained in the varied subject matter he intends to teach.

Whatever attack the student and his supervising teacher choose to make on the problem of selecting the best mathematics, their choice should be referred to a well-developed set of criteria. The following questions are suggested for such use:

1. Is this new learning suited to the pupil's maturity, previous experience, and capacity?
2. Does it lead to other learnings, both mathematical and otherwise?
3. Is it likely to promote the growth of the reflecting, reasoning, and critical powers of the learners?
4. Is it something more than just another trick?
5. Are the processes involved in this new learning economical and direct?
6. Are they subject to verification and check?
7. Will this new learning increase the pupils' understanding of the community?

The foregoing criteria are not intended to replace such practical considerations as, "Will this new work fit into the prescribed course of study?" Rather, they are meant to supplement existing controls, whatever they are.

V. How to Teach Live Mathematics

A number of issues pertaining to the teaching of mathematics were raised in Section III. Several of these dealt with the *way* teaching is done. We now take up a further discussion of such issues which is intended to serve the student teacher at the time he is observing the supervising teacher's work and making plans for his own teaching.

No "best" way of teaching mathematics is offered, only a description of several ways. One of the first questions to arise usually concerns the use of textbooks. Many mathematics classrooms are largely dominated by an adopted text. Under such conditions, the beginning teacher must decide how closely he will adhere to the text. Will he adopt for himself and his pupils the wording of definitions, the context of topics, the structure of proofs, and the illustrative exercises provided in the text? Or will he use the text as something to be supplemented and extended, as a map of the subject, so to speak? Will he and the learners, even from the first, plan to take a hand in making new exercises, new proofs, new explanations? One text-writer, in support of the latter view, urges that his text be used "as a guide post and not a hitching post." By saying this he gives support to teaching which is enriched by a diversity of mathematical materials contributed by everyone concerned, both teacher and pupils, as well as by authors of texts.

Other questions have to do with the way the teacher chooses to arrange groups of pupils. Some schools make an effort to classify mathematics pupils according to ability.[51] In the interest of efficiency

[51] See *High-School Methods with Superior Students* and *High-School Methods with Slow*

each of the several classes assigned to a teacher is selected so as to make uniform procedures practicable. In such classes the same topic is studied by all the pupils at one time. In other schools, the group is heterogeneous, either by accident or design. Pupils of high-grade mathematical ability and superior previous training are placed alongside others of low ability and poor preparation. Obviously such arrangements call for methods considerably different from those appropriate to more homogeneous groups. Moreover, there is considerable doubt whether such a plan can produce good results.

In classes of the first (homogeneous) type, the techniques used may consist largely of lectures, explanations, recitations, out-of-class assignments, quizzes, and a final examination. Such procedures are so common that many student teachers might ask: What other way would there be to handle a mathematics class? It is because this kind of teaching is widespread that the movement for homogeneous grouping has gathered force. Organizing classes of students of about the same abilities and rate of learning is a practice which is favorable, perhaps necessary, to the success of teaching in terms of the entire group. Under such conditions, the teacher can make the assumption that all have an equal start. He then expects the pupils to keep in step from day to day, to reach approximately the same goals at the same time, although in actual practice this ideal is not always reached.

In the second type of classroom, somewhat different plans must be made. If all are to be kept together, the pace must be determined by the abilities of the pupils of average, or slightly-less-than-average, capacity for mathematics. Much out-of-class remedial work must be provided for those of meager capacity, or else the greater part of recitation periods must be devoted to them. Of course, a third alternative would be to gauge the work to the superior pupils and let the less gifted ones suffer failure, the natural consequence of their disabilities.

When faced with such hard choices as these, the student teacher asks, Are there no ways of teaching mathematics other than by the group recitation method?

Umstattd[52] discusses eight plans, five of which offer interesting possibilities for the mathematics teacher, namely: the *Problem Method*, the *Winnetka System*, the *Dalton System*, the *Group-Study Plan* and the *Morrison Plan*.[53]

" 'The Problem Method' is an application of the unit idea. . . . "[54] It involves cooperation among all members of the group. It may concern a topic selected from a textbook, or some question that has arisen in the lives of the students. It might be a problem related to physics, such as how to find the velocity of a falling body or the time required for sound to travel a given distance. Or it may relate to statistics, such as how to select and compute an average which is representative of the sizes or incomes of the families living in a certain area. The method involves the steps described by Dewey[55] in tracing an act of thought. First, the problem is defined. Second, the information is gathered. Third, the information is organized. Fourth, possible ways of finding a solution are listed. Fifth, the most promising means of solution is selected and applied. If it is successful, nothing remains to be done; if not, some of the steps must be retraced.

The Problem Method, offers a broad frame of reference which might prove profitable. If the problem undertaken is

Learners. National Education Association Research Bulletins, Vol. XIX, No. 4, 1941, and Vol. XXI, No. 3, 1943 respectively. Price, 25¢.

[52] J. G. Umstattd, *Secondary School Teaching*, Ginn and Company, 1937, pp. 147–175.

[53] H. C. Morrison, *The Practice of Teaching in the Secondary School*, The University of Chicago Press, 1926.

[54] J. G. Umstattd, *Secondary School Teaching*, Ginn and Company, 1937, p. 147.

[55] John Dewey, *How We Think*, D. C. Heath and Company, revised edition, 1933.

comprehensive, it will give opportunities for pupils of all types to share in its solution, thus making provision for individual differences in the unselected group.

The Winnetka System offers interesting possibilities. It classifies mathematics as a "common essential." As such, it is one of the subjects taught individually, with little concern for the group. Each pupil selects (or accepts) a goal. When he has completed certain exercises he submits to a diagnostic test. If the test reveals weaknesses he is given a self-corrective practice book as an aid to overcoming them. Meanwhile the teacher aids and encourages him in individual conferences in the classroom and elsewhere. When he and the teacher agree that he is ready, he takes the mastery test. The pupil checks his daily work, but the teacher checks the mastery test.

In order to make such arrangements possible, the teacher identifies the goals in successive mathematical topics, prepares practice materials (using exercises of his own making, or taken from available texts), and devises diagnostic and mastery tests. Verbal explanations are made to small groups of pupils who are ready for new sets of goals, rarely to the entire class. To take care of such diversified activity as this plan promotes, the schoolroom is provided with mobile chairs. Then the pupils who are doing approximately the same work are grouped in one part of the room where the teacher can discuss a topic with them without disturbing others who may have mastered that topic or who are not yet ready to consider it.

Procedures such as the Winnetka System give the teacher opportunities to employ all his skill and energy. Whether the student teacher has first-hand experience with such teaching or not, he realizes that it calls for a great deal of planning and preparation. The knack of making it work is not acquired overnight. In any scheme of this kind much depends upon who is the teacher.

The Dalton System is like the Winnetka System. In this system, mathematics is designated as a "major subject."[56] The unit of instruction is a contract of a month's work, subdivided into weekly and daily portions. The mathematics teacher who makes use of this plan follows about the same steps as if he were adapting the Winnetka System to his and his group's purposes. Instead of organizing the individual contracts by topics, however, he organizes them by "months" which represent one-ninth or one-tenth of the entire work of the school year. Both Winnetka and Dalton Plans include all subjects and all pupils in a given school. However, they may be adapted, and successfully, by mathematics teachers in schools where neither plan is used to teach other subjects, without disturbing the work of other teachers. Sometimes a supervising teacher who does not ordinarily attempt to conduct his work along such highly individualized lines will offer to cooperate with his student teacher in giving one of these plans a trial, at least in modified form.

The group-study plan is a scheme for homogeneous grouping within classes, developed by Maguire.[57] The pupils are asked to subscribe to a creed which emphasizes pupil responsibility, perseverance, thoroughness, verification of results, and mutual assistance in learning. The class is divided by the teacher into groups, whereupon group leaders are selected. When the teacher moves from one group to another, the group leader continues raising questions and leading discussion. (The position of leader rotates from pupil to pupil until all have held it during the year.) The discussions center around study charts which are prepared at the beginning. These charts set forth points which were developed when the topic was first introduced. Each is preserved for the special use of the group for which it was made, during the conduct of the unit.

[56] This corresponds with its classification as a common essential in the Winnetka System.

[57] Edward R. Maguire. *The Group-Study Plan*. Charles Scribner's Sons, 1928.

Some mathematics teachers have found the group-study plan well adapted to their needs. The motivating devices (pupils' creed, study charts, leaders' responsibility) are good if wisely handled. The means provided by such a plan offer interesting possibilities for carrying on activities on different levels, all at the same time.

A fifth plan worthy of consideration is Morrison's.[58] It is a comprehensive, well-developed view of the educative process. Morrison refers to it as "systematic teaching." In *The Practice of Teaching in the Secondary School*, Morrison analyzes what he terms "operative technique" into the following steps:

1. Listing the units to be mastered.
2. Identifying the objectives of each unit.
3. Applying to each the "mastery formula" of *pre-test, teach, test, adapt the procedure, teach and test again to the point of actual learning.*

Thus his method centers around the unit which he defines as a "significant and comprehensive aspect of the environment, of an organized science, of an art, or of conduct, which being learned results in an adaptation in personality."[59] Mathematics teachers are particularly favorable to his views concerning mastery. Mathematics is a field of knowledge in which the contrasts between learning and non-learning are sharp. Vague, half-defined concepts are often worse than useless in mathematics. Furthermore, the mathematics teacher likes Morrison's thesis that true learning is permanent. Here is something worth working for: to teach so well that the essentials of what has been taught will never be lost; not only this, but to teach in such a way that *all* pupils attain all the important goals.

Once the appropriate unit is selected, the teacher proceeds to formulate its specific objectives. Then he prepares a "guide sheet" and testing materials.

The objectives fall into three classifications, namely, special abilities, attitudes of understanding, and attitudes of appreciation. Many mathematics teachers have found it helpful to list their specific objectives in writing, always in terms of the pupils' learnings or "adaptations."[60] A helpful device is to state those having to do with special ability using the words, "the ability to . . . , " or "to be able to. . . ."[61] Attitudes of understanding and of appreciation are expressed, "An understanding of . . . " and "An appreciation for. . . . "[62]

At this point it is well for the student teacher to remind himself that the crucial objectives for a mathematics unit are the attitudes of understanding. On the other hand, special abilities (skills) are necessary to mastery, to use Morrison's terms. There is no real conflict between the two, yet the tendency is to teach skills as though they were the one thing needful, to the neglect of understandings. Such experience is thin and inadequate. No matter how nimble pupils become in mathematical processes, such skill can never be substituted for reasoning. As Whitehead has put it,[63]

> Without a doubt, technical facility is a first requisite for valuable mental activity: we shall fail to appreciate the rhythm of Milton, or the passion of Shelley, so long as we find it necessary to spell the words and are not quite certain of the forms of the individual letters. In this sense there is no royal road to learning. But it is equally an error to confine attention to technical processes, excluding consideration of general ideas. Here lies the road to pedantry.

Morrison would have both special abilities and understandings taught directly, by means of appropriate exercises. Mathe-

[58] Henry C. Morrison, *The Practice of Teaching in the Secondary School*, The University of Chicago Press, 1926.

[59] *Ibid.*, pp. 24–25.

[60] Morrison defines an adaptation as a "qualitative accretion to personality." *The Practice of Teaching in the Secondary School*, The University of Chicago Press, 1926, p. 74.

[61] This is preferable to "Have the students learn . . . . " To make such a distinction is a little thing perhaps. Nevertheless experienced teachers have found that it sets them on the right path to begin thinking and writing about the unit in terms of the pupils and their activities rather than in terms of the teacher and what he expects to do.

[62] See pp. 111–112 for illustrative statements of objectives.

[63] A. N. Whitehead, *Introduction to Mathematics*, Oxford University Press, 1911, p. 8.

matical appreciations, while no less important, are not engendered by the same means. Such attitudes are *caught* rather than *taught.* The teacher gives evidence of his appreciation of mathematics by his own sincerity and devotion. He is constantly striving to extend his own knowledge of the subject. He creates an atmosphere in his schoolroom favorable to mathematics by displaying pictures of great mathematicians, models and instruments. He himself cultivates traits appropriate to a mathematician, such as open-mindedness, reasonableness, honesty, reliability, objectivity. Thus he induces pupils to hold to these values by example, a means of teaching which loses nothing by being indirect.

The guide sheet used in teaching the unit may be simple or detailed. It provides what Morrison calls the assimilative materials. Blackboard instructions may be used in lieu of mimeographed sheets furnished to individuals. In addition to the elements of the unit, the instructions should include references to textual materials; also suggestions for special projects to be undertaken voluntarily by pupils.

Three kinds of tests are prepared: the pre-test, the assimilation tests, and the mastery test. The pre-test is given simultaneously to all pupils in the group. It covers both the learnings which must be attained before undertaking the unit, also some of the learnings included in the unit. It is important to know if pupils are ready, also to know what part, if any, of the proposed undertaking is familiar to them. (The guide sheet cannot be fully completed until the results from the pre-test are tabulated.) The pre-test might show that some individuals have mastered the unit already, that no further work is needed unless it be in the direction of special projects or higher degrees of skill. The assimilation tests are directed toward successive stages in the unit. They are planned before the unit is presented but, like the guide sheet, they are subject to changes while the unit is being assimilated. Their purpose is to test the quality and extent of the learner's progress toward unit goals. Assimilation tests may be given to all pupils at one time or they may be given to small groups at various times, depending on their rates of progress. The mastery test is comprehensive. It is made in terms of the original objectives. If it is valid for those objectives, then the ability to respond successfully to the final test denotes that mastery—permanent, true learning—has been reached.[64]

In mathematics classes, the mastery test is best given at the time an individual pupil applies for it. If he fails it, both he and the teacher regard it as an assimilation test. The pupil returns to the guide sheet and his studying until such time as he and the teacher feel that a second trial is in order. Hence to provide individual pupils with re-trials, the teacher must be adept at making multiple forms of the mastery test. In every class a few pupils

[64] According to Morrison, this level is not the highest to be attained. Mastery means that *essential* understandings, abilities, appreciations will never be lost, even though very little memory of details and little skill may remain. To illustrate: The learner who had mastered a unit in logarithms sufficiently to pass the mastery test, in later years might remember little more than, say, the following facts: (1) for logarithms to the base 10, mantissas remain the same for successive multiples of 10. (2) Characteristic increase or decrease, as the case may be, as antilogarithms are multiplied or divided by 10. (3) To multiply one number by another logarithms are added. He might lose all his skill in reading mantissa tables quickly and accurately, in making interpolations. These and other learnings might escape him, yet he has been "permanently changed" in that he appreciates the uses of logarithms for certain kinds of mathematical work, whereas before he studied logarithms he had no such appreciation. Also, he knows such things as *how* to use mantissa tables; he understands the functional relationship between antilogarithms and logarithms. Before "mastering," he had no such understandings. On the other hand, this level of mastery is not sufficient to enable him to step into the chart house of an ocean-going vessel and find the ship's position in a dependable and economical manner, by the use of logarithms. What it does give him is a definite basis on which to build the refinements of logarithmic technique whenever he finds it necessary or desirable to do so, as in navigating a ship.

of outstanding ability will be ready far in advance of the others. They may take the mastery test together. A few days later, the main body of the group will be ready. After them, of course, come those who, for good reasons or bad, are late in mastery. Before the last of these take the test, both they and their teacher must do some concentrated work.

In the foregoing discussion of tests it is implicit that the Morrison unit is not a restricted, day-to-day, piece of learning. It is "comprehensive and significant." It requires time to master it. How long? No one knows. In effect, Morrison says that the learning product is a constant (i.e., everyone either has it or has it not) and time is a variable.

The teaching of the unit is marked by five phases, *exploration*, *presentation*, *assimilation*, *organization*, and *recitation*. To each phase is given the appropriate allotment of time; a class session or so each to exploration and presentation; a number of sessions to assimilation; and one or two each to organization and recitation.

In most mathematics classes the exploration phase consists of administering the pre-test. This done, the teacher presents the unit, employing the greatest skill of which he is capable. At that time, his purpose is to point out the objectives in such a way as to induce (motivate) pupils to make the effort necessary to reach them.

After the presentation phase, the classroom becomes a study room. The teacher prefers, especially during early stages, that all study be under his direct supervision. Therefore he opens the room to pupils during as much of the day as school facilities will allow, for extra-class work on the unit. Wall spaces are hung with charts and formulas. Bulletin boards are used to display clippings from current publications, special reports, and other aids to learning. The regular class periods are also for study during this third phase. The guide sheet serves to direct and control the work. The teacher moves from individual to individual and keeps himself informed of the progress of each pupil. Sometimes a group will gather around him to have him clear up some misconception that has emerged. From time to time, he gives short assimilation tests. The data from such tests guide his efforts to assist the pupils. Such is the assimilation phase.

Soon the more able pupils begin to give evidence of mastery. When they have passed the mastery test, the teacher discusses with each the possibility of his undertaking one or more of the individual projects suggested in the guide sheet, or some other project related to the unit.[65] Or it may be that the pupil has already begun work on some such voluntary undertaking. As soon as he has passed the mastery test, he begins to give all his time to his special contribution to the unit. He is released from coming to the classroom during regular periods, in many instances, so that he may make uninterrupted use of shop, library and community resources.

Finally all have reached the point of being able to pass the mastery test. Now the room is cleared of all its aids to study. Books, papers, charts, tables, graphs, instruments are put away. The pupils who are working on voluntary projects are recalled. The entire class meets for the organization phase. Very frequently a pupil will participate as chairman. The group reviews the entire unit, attempts to relate it to the rest of the field of mathematics, and evaluates it in terms of its cultural, disciplinary, and utilitarian values. The pupils now see the unit somewhat as the teacher saw it when he presented it to them. They make an outline cooperatively, using some such headings as: *Which of our previous learnings have been enriched or made clearer by this unit? What are the most important goals in this unit? How might we have mastered this unit more*

[65] Conceivably, the pupil might undertake some project not directly related to the unit, but having mathematical, or scientific, or social implications. The teacher is more than a mathematics instructor in the narrow sense.

thoroughly and economically? To what new goals does this unit point? The resulting outline or brief is added to the pupils' notes as a record of the entire experience. The pupils have organized the unit.

In this connection, Morrison advances the thesis that there are three elements in the learning cycle, namely: stimulus, assimilation, and response. He believes that learning is incomplete unless it has progressed through all phases of the cycle, which means that the learner must *do* something with his new powers. The organization phase provides for certain responses to the learning situation, likewise the recitation phase which follows. Thus they serve to round out the learning cycle.

During the recitation phase, the class becomes an audience while selected pupils make reports of their projects or of other experiences associated with the unit. Pupils are called on in turn to make these oral presentations, two or three for each unit. The others write their reports.

After this final meeting for the recitation phase, the class is ready for a new unit. All have mastered. None has been rushed or neglected because of another; none has been forced to mark time. The teacher closes the unit by writing the record of each pupil's achievements in the individual folder kept for that purpose.

In view of the widespread attention which has been given to Morrison's plan, it is likely that the foregoing discussion will offer but little that is new to the student teacher. However, it has been set forth in some detail because it offers a promising technique for teaching mathematics, one in which provision is made for individual differences without sacrificing the benefits that result from group discussions and shared activities.

In choosing a teaching plan, the student teacher should bear in mind that he must be thoroughly familiar with that plan. To succeed with any but the most simple procedures, either he must have lived with them (as a pupil or observer, or both) or else he must have strong guidance from his supervising teacher. As was the case with subject matter, he must choose a plan with certain definite considerations in mind. The following are suggested.

1. Have I a clear notion of the plan I propose to use?
2. What are the strong and weak points of this plan?
3. Is the proposed plan one to which the pupils are likely to make a favorable response?
4. What risks are involved in this plan, in terms of my ability to maintain good working conditions in the classroom?

In addition to studying plans, the student teacher must consider some general principles pertaining to mathematics teaching. One of these is what might be termed the principle of initial random movements. The student teacher must not forget his own early experiences with mathematics. Accordingly, he must learn to be tolerant of pupils' early difficulties. As Mursell says:

> The actual building of mental grasp and organized power in . . . problematic thought is always . . . a highly disorderly process marked by widely ranging trial and error and experimentation.[66]

Also he warns that:

> Some teachers tend to oversystematize and schematize the pupil's attack on technical thought problems.[67]

Another such principle might be called the principle of deferred drill. Mathematics teachers need to guard against trying to fix habits prematurely. Again according to Mursell:

> Extensive repetition has its place late in the sequence of . . . learning, for its function is to consolidate and strengthen a pattern of response and perception which is already formed.[68]

Another principle is implied in Professor Dresden's discussion of "shoes and stockings."[69] In mathematics certain operations

[66–68] Reprinted from *The Psychology of Secondary School Teaching* by James L. Mursell, by permission of W. W. Norton and Company, Inc. Copyright, 1932, by the publishers.

[69] Arnold Dresden, "Mathematics and the Social Sciences," THE MATHEMATICS TEACHER, XXXII, December 1939, p. 340.

must follow certain others just as shoes must follow stockings when dressing. An example of this is found in equations in which both addition and multiplication are indicated. The multiplying must be done before adding. With hats and coats it is different; either one can go on first. So it is with a series of addends: It makes no difference in the final outcome which ones are added first. Teachers must see that such distinctions as these are made explicit.

A further principle concerns the surroundings in which mathematics is taught. This applies both to the physical and the social environment. Very often mathematics classrooms are barren places.[70] The student teacher does what he can to make his room as favorable to learning as possible. In this he remembers that his stay is temporary, that he must use restraint and tact. Also he must not overlook the social resources. For the student teacher to tell his classes about the uses of logarithms in engineering is one thing: it is another thing for him to invite an engineer to come to class and relate his experiences with logarithms, to answer pupils' questions directly. Some mathematics teachers make valuable use of the community by introducing pupils directly to a wide range of interesting personalities and activities.

Still other principles will emerge as the student teacher proceeds with planning, observing, and teaching. A good measure of the value of his experience in student teaching is the extent to which he becomes aware of the forces which conduce to learning. Then as he becomes aware of these forces. ways of putting them to work will emerge.

[70] See "Equipment of the Mathematics Classroom," Fifteenth Yearbook of the National Council of Teachers of Mathematics, Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, *The Place of Mathematics in Secondary Education*, pp. 238–245.

VI. What Are the Phases of the Student Teacher's Experience?

The first phase of the student teacher's experience is observation. He takes his place in the mathematics classroom and begins to "get the hang of things." Here is where he will have an interesting and valuable experience, extending over a number of weeks, perhaps an entire semester.[71] He watches the pupils, the teacher. He takes stock of the room. Soon, out of the general pattern certain features begin to emerge. He notices the way in which routine matters are handled, such as attendance, lighting, ventilation, equipment, seating. He watches the pupils to see how they respond to the situation. He begins to make tentative plans for his own teaching.

Many beginners have found it helpful to make what might be called controlled observations. One device for doing this is to take a sheet of paper having a line dividing it vertically in halves. On one side the observer makes notes of all the teacher's activities, his questions, his work at the board, his assignments, and the like. On the other side he records the pupils' activities. This memorandum later becomes the basis for a conference between the experienced teacher and the observer. It may be helpful to both. In this way the beginner becomes more sensitive to the problems of teaching. Because he cannot record every activity, he begins to evaluate what is going on, in order to select the most important activities for his notes. The teacher whom he has observed gains by having an objective account of his teaching.

Another device is to have the observer

[71] The arrangements as to time differ in various institutions. Teachers College operates two plans, one called *Practice Teaching*, the other *Interneship*. The first plan calls for a semester's experience with teaching; the second for an entire school year, or two semesters. New Jersey State Teachers College at Montclair provides twelve weeks of practice teaching. Some schools divide the time into three weeks each of observation, participation, and teaching.

report on certain points such as the following given by Sanford, Habberton and MacHarry in their *Manual for Practice Teaching,* published at the University of Illinois.[72]

1. Classroom routine
2. The assignment
3. Questioning
4. Objectives, learning exercises
5. Specific methods
6. Problems of personal adjustment
7. The textbook
8. Supplementary and enrichment materials
9. Evaluation

Knudsen[73] offers a device for observers which might prove valuable for the student of teaching. He suggests grouping teachers' questions or challenges to pupils under three headings, *basal learning, diagnostic,* and *test questions.* To make the analysis necessary to group the teacher's questions under these categories provides a challenging experience to the one making the analysis. Because it requires maturity and skill to use Knudsen's device successfully, the record resulting from a beginner's use of such a scheme might have little intrinsic value.

Drewry[74] has developed a scheme for studying pupil activity, a graphic seating plan record. The observer takes a sheet of paper on which horizontal and vertical lines have been drawn, making rectangular cells in rows, six or seven to a row, and six or seven rows, depending on the number of seats in the classroom being visited. This, of course, resupposes that the seats are arranged in straight rows. If so, each cell corresponds to a seat. The individual pupils' activities are recorded in the cells by means of symbols. For example, if a pupil should volunteer an answer to a question, that fact would be recorded by a "v" put in the cell whose location on the paper corresponds to the location of his seat in the room. An answer drafted by the teacher would be recorded by the symbol "a." This device has obvious limitations for mathematics observations, since it breaks down when the pupils leave their seats for the blackboard. However, it does offer one outstanding advantage: Whenever the teacher is conducting a question-and-answer session—and such sessions are frequent in some mathematics classrooms—there is an advantage in having a record of responses. Among other things, the teacher may find from this record that he is giving all or almost all of his attention to certain pupils, to the neglect of others.

Back in the nineteenth century the students at the Worcester (Massachusetts) State Normal School kept diaries of their experiences in the schoolroom. The neophyte might profit by reading some of these anecdotal records as published by Gordy.[75] Because less controlled than the devices previously described, such informal records may be more valuable both for the practice they give in critical observation and for the data gathered.

Student teachers at Teachers College have used broad frames of reference for observation of mathematics classes. At one time the members of such a group were asked to report what they saw in terms of mathematical aims. They classified activities in terms of *utilitarian, disciplinarian,* or *esthetic* aims. The notes they took in doing so were used as bases for conferences and seminar discussion.

Further frames of reference for observers are suggested by the following questions:

1. Is the teaching being done in terms of comprehensive wholes or in isolated bits?
2. Does the teaching emphasize repetition and drill, or initiative and understanding?
3. Is the main concern for the product or for the process of learning?

[72] Charles W. Sanford, William Habberton, and Liesette J. MacHarry, *Manual for Practice Teaching,* Stipes Publishing Co., Champaign, Illinois, 1939, pp. 11–13.

[73] Charles W. Knudsen, *The Evaluation and Improvement of Teaching.* Odyssey Press, 1932.

[74] Raymond G. Drewry, "A Study of Pupil-Teacher Participation in the Classroom," *Los Angeles Educational Research Bulletin,* February 1927, 6: 9–10.

[75] J. P. Gordy, "Rise and Growth of the Normal School Idea in the United States." *Circular of Information,* U. S. Bureau of Education, 1891, No. 8, pp. 77–80.

If the student and his advisers care to extend their lists of such questions, the following might prove acceptable. They were suggested by a short story in which, strangely enough, the hero was a mathematics teacher.[76]

1. Does the teacher teach definite things?
2. Does he teach how to *use* formulas, not just tricks?
3. Does he make mathematics "not a task, not drudgery, but rather at the one time an intensely practical method and tool, and an imaginative dream, a wild conjecture"?

In many cases, observation seems to become more valuable as arrangements and viewpoints are made more flexible. At any rate, here is a point which must be decided, whether or not observations shall be subject to specific controls. Whatever the decision, one set of controls must never be ignored, namely, the ethical considerations involved.

There are many excellent statements available concerning the ethics of observation. Basic to all of them are consideration for the welfare of pupils, of the teacher being observed, and of the observer. The needs of the pupils must remain uppermost. Accordingly, a sound principle seems to dictate that demonstration lessons, intended to present some technique of teaching, must be planned with the greatest care. The natural sequence of learning must not be interrupted. A presentation must not be introduced prematurely or belatedly on account of the observer's convenience unless it is the considered judgment of the teacher, the one responsible, that the group is not likely to lose by such shifting. A second principle has to do with the observer's personal conduct during his visit. He must be as unobtrusive as possible. To accomplish this he must arrive on time, seat himself quietly in an inconspicuous place, refrain from talking or other overt behavior.[77] (In campus schools, teachers and pupils are accustomed to being observed. Consequently in most cases the visitor would have to be guilty of rather marked thoughtlessness to disturb them.) Further, the observer should make it a point to remain until the class session is over. A third principle concerns the relations between the observer and the teacher observed. The observer must avail himself of every means of gaining an intelligent, sympathetic understanding of the teacher's purposes, of the pupils' capacities, and of the general situation. He should seek interviews with the teacher before and after observing. If he takes notes during observation, he must if practicable offer the teacher a copy. He should seek to get the teacher's interpretation of what is going on. A fourth principle has to do with the observer's relation with the profession of teaching. He must refer all his actions, both in the classroom and out, to the rights and needs of others. Even though, up to the time of his visit, he may have thought of himself as a comparatively irresponsible person, he now is vested with the serious obligation of considering the effect of anything he may say or do relating to the school, subsequent to his observation. Careless remarks about the quality of the instruction he has witnessed are gross violations of the teacher's hospitality. Little as such remarks deserve it, they sometimes have unexpected destructive force. Unfortunately, it is much easier to tear down than it is to build up. To give practice in the use of this principle, the suggestion is sometimes made to student observers that they record "what they liked best."

When referred to sound educational principles and combined with intelligent conferences and seminars, the observation phase of student teaching becomes a valuable part of the future mathematics

[76] Edward L. McKenna, "A Vacant Chair," *The Saturday Evening Post*, Vol. 213, No. 16, pp. 14–70.

[77] One director of training is quoted as insisting that observers maintain a "poker face," devoid of expression. However, this kind of demeanor might prove almost as annoying to teachers and pupils as would extreme animation on the part of observers.

teacher's experience. It leads naturally to the next phase, participation. Here it may be said that observation after some experience in teaching is much more profitable than the initial observations.

By the time this second phase has been reached, the student has some knowledge of the pupils and what they are doing. He is ready to adopt a more active role. The supervising teacher gives him some responsibilities. In many mathematics classrooms enough rulers and compasses are kept on hand to serve each pupil. The student may take charge of these and other articles of equipment. He may take over the attendance records, the ventilation and lighting, the care of blackboards, and other incidental duties.

Many student teachers in this phase find opportunities of contributing some useful article of their own making to the mathematics classroom. If the student has the facilities, he may make a portable cross-section blackboard for use in graphical representation. Or he may make some models to be used in visualizing mathematical relations or processes.[78] He may arrange an exhibition of regular solids. On the third floor at Teachers College there is an extensive and permanent collection of such models and solids. These may serve to suggest a wide range of possible contributions. Better still, he may offer to assist some of the pupils in such undertakings. By doing so both pupils and student teacher will be benefited.

During the participation phase, the student teacher begins to read pupils' mathematics papers. He is encouraged to analyze their difficulties and to point out characteristic errors. A little later, the supervising teacher will give him an opportunity to teach small groups of individuals who seem to need special encouragement, additional explanations. This is work which is called "remedial work" in pedagogical terminology. The breaking down of a mathematical pattern into its component parts is a highly individual affair. The experiences that will enable one pupil to "differentiate," as the psychologist would say, are often not the same experiences which will lead another to such insights. Therefore pupils need individual attention. The student teacher gains through his experience with teaching pupils who have special difficulties.

Marking test papers and organizing the data therefrom is an activity undertaken by the student teacher in the participation phase. A good device to use is that of writing pupils' names in a column at the left of a piece of cross-section paper, then listing each exercise used in the test across the top. As each paper is marked, an appropriate check is put opposite the pupil's name and under each of the exercise numbers showing whether or not he succeeded with the several exercises. When the record is complete, totals can be written opposite the pupils' names, also under the exercise columns. Thus at a glance the supervising teacher can see how many exercises were completed by each pupil, and how many pupils completed each exercise. Such data are indispensable for reteaching.

While in this phase the student teacher more than ever is looking forward to the time when he will exchange places with the supervising teacher and be the observed rather than the observer. In his conferences with the supervising teacher he has begun to discuss plans for the next phase of student teaching, that of actually assuming responsibility for the entire class.

Among other things to be decided concerning this next phase of student teaching will be:

1. The type of organization to be used, whether one of the unit plans discussed in Section V, or somewhere else.
2. The topic to be taught.
3. The approximate time he is to begin teaching.

As soon as these general points have been discussed, the student teacher begins to formulate objectives or goals to be reached

[78] For models and other multi-sensory aids to teaching see the Eighteenth Yearbook of the National Council of Teachers of Mathematics.

during his teaching, to select and devise exercises and tests to be used. These he will submit to his supervising teacher for criticism and amendment.

At this point the student begins to understand as never before why men and women like to teach mathematics. As soon as he begins to organize and clarify his own thinking in anticipation of teaching, he realizes more than ever what possibilities there are in such an adventure. If he is fortunate in his supervising teacher he will find his initiative, enthusiasm, and imagination growing under wise encouragement. If he has a real love for mathematics, as indeed he should have if he is to teach mathematics, then both he and his supervising teacher will look forward with pleasure to their conferences, to the plans they can work out. As the time approaches for the student to begin teaching, they will spend more and more time preparing for it, discussing the value and the meaning of the mathematics to be taught.

Finally the time comes for the student teacher to begin. He has listed objectives, prepared assignments (either in the form of a guide sheet or in some other form), checked on necessary materials.[79] He has thought of an appropriate way to start. All these things have been discussed with the supervising teacher.

In many schools it has been found advantageous for the supervising teacher to say a few words to the pupils, announcing that Mr. Doe is to be the teacher for several weeks, and asking that they cooperate with him in making their work successful. (This may not be necessary, for the student teacher has been checking attendance, assisting individuals and small groups, marking and returning test papers, and attending to many other duties, all of which has made him a familiar figure to the group.) After the student has begun, the supervising teacher leaves the room or retires to an inconspicuous position while the student teacher proceeds.

If the student thinks that there may be pupils in the group who do not know him, he may write his name on the blackboard. Good taste and judgment will limit him in making further introduction of himself. If, for example, his topic is logarithms, he may proceed at once to administer a simple pre-test. He may explain his purpose in giving the test, namely, to enable him to gauge the ability of the class to handle exponents, and the various other abilities necessary to mastery of the new learning. Whatever his plan he must stand ready and willing to modify it, from the first day on, to meet unexpected developments.

A good deal has been written about the student teacher's poise, personal appearance, voice, and manner. It is our opinion that it is easy to say too much about such matters. The student teacher usually has poise because he is physically strong, emotionally equipped to meet the strain of new situations, familiar with the personnel of the class through previous participation, and thoroughly prepared by previous study to do the work at hand. It is the joint responsibility of the student teacher, the supervising teacher, and the directors and coordinators of student teachers to see that he has all these traits. It is true that he has never had full charge of a mathematics class before, but, unless his previous experience has been most unusual, he has spoken to groups the size of the average class. Thus he has some basis for judging as to how he can use his voice effectively. By making his manner sincere, direct, informal, without risky attempts at humor, he launches himself into this new phase of student teaching without fuss and undue strain either to himself or to the pupils.

The good supervising teacher exercises great restraint at this stage of the work. He withholds all unnecessary adverse criticism. In conference he points out and

[79] For example, if he is going to teach a unit on logarithms, he will want to be sure that mantissa tables are at hand. Also he may want to provide some logarithmic graph paper.

thereby emphasizes the good features of the beginner's teaching. Above all, he does not interrupt in class. There are a good many don'ts for supervisors to observe. Among them are:

1. Don't hover too closely ("to be discreetly absent . . . is definitely a part of the job of supervising").[80]
2. Don't be a perfectionist. (Like everyone else, the mathematics teacher must learn by doing, and this is the student teacher's first try.)

Probably the all-time low point in supervising was reached when a mathematics training teacher sat at the back of the room during a class and wrote notes to nearby pupils, requesting them to ask the student teacher "catch" questions. Fortunately the student teacher in that situation was a young women of iron nerves and monumental poise or she would have withered under such an assault.

Reeve[81] gives a list of student teachers' faults which was offered by student teachers themselves at the University High School of the University of Minnesota. The list is pointed definitely toward the class situation in which the entire group is considering some topic or problem. It is of especial interest because it grew out of student teachers' observation of each other's teaching:

1. Asking leading questions.
2. Repeating the answers of the pupils.
3. Accepting untidy and unsatisfactory work.
4. Overlooking small errors in oral and written work.
5. Hurrying pupils too much when they are trying to reply to questions.
6. Calling on the brighter pupils too much instead of distributing the questions.
7. Over-attention to dull pupils when it is clear that the time is being wasted.
8. General tendency to say "good," "yes," etc., to dismiss a pupil after he has recited.
9. Saying "all right" after a pupil has recited when "that will do" is really what is meant.
10. Asking questions the answers to which are impossible because of lack of maturity of the pupils.
11. Unintelligent use of words and phrases as "cancel," "transpose," and "throw everything on the left."
12. Carelessness in putting clear and pointed questions, i.e., the questions are often poorly worded and indefinite.
13. Talking and working too fast, i.e., getting ahead of the class so that they cannot think as fast as the teacher talks and works.
14. Calling the name of the pupil first before setting the question and thus encouraging inattention of the other pupils in the class.
15. Asking questions too rapidly, especially if the pupil does not answer quickly, instead of giving the pupil time to think.
16. Helping the pupil too much, i.e., doing too much of the work that the pupil should do in order that he may be made to feel responsible.
17. Accepting from pupils answers that are not clear by interpreting their meaning, and thus robbing the rest of the class of the knowledge of the correct answer.

In some situations, student teachers are given a great many opportunities to analyze their work and their personalities, even from the beginning of their teaching periods. This may be a mistake. Let the beginner develop his pattern of teaching before he begins to break it down, to judge it in detail. To some, it seems reasonable to suppose that good teaching is built up by studying and mastering one trait after another. However, the evidence runs counter to such a theory. Learning and growth do not take place in that way. As Mursell says, "The actual building of mental grasp and organized power . . . is always . . . a highly disorderly process marked by widely ranging trial and error and experimentation."[82]

The student teacher has seen the supervising teacher at work, as well as many other teachers. He has become sensitive to teaching problems as a result of his observation, previous study, and conferences. He visualizes himself in the teaching role. Yet many things which escaped him while he was an onlooker now clamor for attention, now that he has begun to teach. Some of these may be:

[80] Anita D. Laton, Chairman, "A Handbook for Student Teachers and the Supervisory Staff," *University High School Journal*, January 1940, Vol. 18, No. 2, p. 89.

[81] W. D. Reeve, *Student Teaching Bulletin*, The University of Minnesota, 1923, pp. 13–15.

[82] James L. Mursell, *Psychology of Secondary School Teaching*, W. W. Norton and Company, 1939, p. 178.

1. When pupils point out errors on the part of the teacher, how shall he react to the criticism? Working at the board in front of groups sometimes brings its own difficulties. Teachers find themselves occasionally making errors then which they do not recall having made at any other time. When these occur, will he be able to control his embarrassment?
2. When pupils do not manifest any interest, what shall he do about it? Never, until now, did the student teacher realize how demoralizing it can be to have to face a bored group. The modern slang expressions, "dead pan," "sourpuss," and the like, begin to take on new and horrid meanings after having encountered such visages in the classroom. Shall he just ignore such behavior, or try to do something to alter it?
3. When his presentations do not "take"—when no attempt of his to make a process clear succeeds—then what?
4. When pupils will not undertake out-of-class assignments, what shall he do about it?
5. What shall he do to restore his own morale when faced by extremely poor test performance on the part of his pupils?
6. What shall he do on the day the pupils just will not get down to work?
7. How shall he handle the pupils who persist in the query, "What good is this topic?" or "What good am I going to get out of studying mathematics?"

There are few, if any, mathematics teachers who have not faced these questions and others like them which concern the pupil-teacher relationship. Yet obviously no ready-made answer can be given to them. However, it can be stated in general that situations when they arise must be marked by mutual tolerance, consideration, respect, courtesy, tact, and sincerity. Fortunately these traits are contagious. When the supervising teacher gives evidence of them in his dealings with the pupils and the student teacher, the pupils and the student teacher reciprocate. When the student teacher, likewise, is considerate of pupils, they take somewhat the same attitude toward the student teacher that they took toward the supervising teacher. Thus, naturally and easily, these same traits mark the new teaching-learning relationship between the novice teacher and his pupils. They learn to "get along," to find ways of resolving their difficulties, of easing awkward situations. A good motto for any teacher is "Be square" with the pupils.

Other questions concern the making of day-to-day plans. If the student is teaching in terms of daily lessons, each of which is thought of as a distinct teaching-learning experience, then the matter of daily plans becomes crucial. Some supervising teachers settle the matter by requiring detailed, written plans to be submitted in advance for criticism and amendment. Once they have been approved, it is expected that the student teacher will adhere to them. Sometimes such a rule seems inconsistent to the student teacher. He recalls that during the observation phase the supervising teacher did not always bring written plans to class for use in his own teaching. He might reason that when plans are made in advance and closely followed, the work tends to become somewhat inflexible; that, after all, plans made in advance are armchair sorts of things. Made for hypothetical situations, they live only in the mind of the planner, like imaginary dialogues. The boys and girls in plans are made of straw, not flesh and blood.

Still, one might reply, does this augur that no plans should be made? No. The master teacher does not go into the schoolroom without plans. On the contrary, he has definite ideas as to goals, activities, and procedures. These may not be written, as frequently they are not, but they exist nevertheless. Furthermore, they are good plans. Out of long experience, he is able to visualize and predict the coming situation. He knows both the children and the mathematics they are to learn. He knows how to make the subject matter come alive by relating it to first one and then another of the pupils' experiences. The beginning teacher does not have these resources. Issues are less clear-cut to him. He has greater difficulty in foreseeing what is likely to happen in the classroom; what the outcomes may be. The written plan is for him a necessary device. It enables him to "think himself

clear-headed" about the coming work.

Then, how prevent the plans from becoming too rigid? Mossman[83] gives an answer to this question in the following statement of the function of written plans. She suggests that they constitute:

1. A definite commitment to writing of what he (the student teacher) thinks will happen. This induces more specific thinking in advance of teaching.
2. A point of reference for the teacher by way of getting his bearings, should class discussion be such as to lead to confusion or consideration of the irrelevant.
3. A record of what he thinks will happen, and, by a simple device of checking at the close of the lesson and noting new points arising, a record of what did happen.
4. A basis of checking accomplishment against the work as originally proposed.

Such plans need not be elaborate. They should include a statement of the specific goals, of the proposed means (exercises, discussions, tests) for reaching those goals, and the ways in which progress is to be evaluated. These day-to-day plans supplement the more complete organization for the entire unit, laid down before he began teaching.

At this point the student teacher realizes more fully the value of the comprehensive plans he made under guidance while still observing and participating in his supervising teacher's work. Now that he is teaching he realizes that the hours he spent listing objectives, making guide sheets (or comprehensive assignments), and preparing tests have not been wasted. He makes notations, corrections, comments on his plans and determines to keep them for future use. He sees how vital it is to have organization and forethought.

If this preparation has been adequate, the work of the teaching phase goes from day to day without undue strain. The student teacher has steeped himself in the subject matter to be taught. He is prepared to discuss special phases of the work with students who desire to undertake individual projects. He is ready for the class sessions with daily plans based on his more comprehensive plans. He has conferences with his supervising or training teacher and with the college supervisor. He feels that he is getting on. Because he wants to succeed with this work, he gives up many outside activities that he might otherwise undertake. Each day he wants to be physically and mentally alert as he approaches his work.

From the beginning of his period of full participation in teaching under guidance, the student teacher is faced with the need for evaluating his pupils' work. He must be able to estimate progress and appraise final outcomes. There are a number of bases for making evaluation. Among these are the pupils' performance on the pre-test, the assimilation test, and the mastery test; also, the informal interchanges between pupil and teacher during classes; the special projects undertaken by some or all of the pupils; and whatever unconstrained behavior of pupils, outside of the classroom, happens to come to the teacher's attention.

When planning to use written examinations, the student teacher does well to study the possibilities of objective tests, both home-made and standardized. In mathematics, all tests are largely objective, in that the pupils' performance is subject to check by any reviewer. Their responses (processes used, findings obtained) must be listed as correct or incorrect, except possibly in the case of mathematical proofs where there might be some variance in judgments concerning rigor. This gives the mathematics teacher an advantage in making tests. He may add to this advantage by using devices in which the manner as well as the nature of responding is controlled. One device used in connection with mimeographed tests is that of directing that the final results obtained be written within small rectangles provided for that purpose on each copy. Then a key device is prepared by cutting windows of the same size in a sheet of paper, located so as to correspond with the several rectangles. Thus, when the key

[83] Lois C. Mossman, "Changing Conceptions Relative to Lesson Planning," *Supervisors of Student Teaching*, 1925, p. 43.

is laid on the pupil's paper, the latter concealed except for the rectangles containing his answers. Above each window is written the correct response which compares with the response appearing through the window. This device makes it possible to check papers quickly and accurately.

In addition to homemade tests, there are numerous standardized and other objective tests for use in mathematics.[84] Norms are furnished with standardized tests which enable the student teacher to compare pupils' performance with that of similar groups or individuals. Care must be taken not to confuse norms with standards. As was stated previously, the standard in mathematics is perfection, although not all may achieve it. To use norms as standards is to set goals that are too low for many in the group.

Formal testing does not provide the only means of evaluation. Informal relationships of the mathematics classroom, pupils' out-of-class work, special conferences, and the like are important means of gauging achievement. Many good teachers keep anecdotal records of individual pupils' behavior. Such occurrences as this are recorded:

> Today Elizabeth D—— was one of the few who were able to solve quadratic equations by means of graphical representation. This success on her part is the result of considerable effort. She came to the class poorly prepared, but she was determined to succeed.

Other teachers attempt to set down their impressions of pupils' progress by recording letter symbols or per cents, from day to day. This scheme is useful provided the letters or per cents are not averaged. It would naturally be possible to overdo the use of daily memoranda, whether anecdotes or symbols.

A third basis for evaluation is found in the pupils' individual projects. What better way could there be of judging the value of an experience than by considering the work they undertake voluntarily? While less able pupils are striving to reach the objectives of a unit, some choose to go on into advanced or related aspects. For example, some might study natural logarithms while their less diligent or less gifted classmates continue to study logarithms to the base 10. Or a pupil might become sufficiently interested in Napier and Briggs to make a special study of their lives and contributions, to be reported later to the entire class. Others might elect to continue work with the slide rule, independent of the class. The student teacher keeps in touch with these varied activities and through them becomes more intelligent concerning the pupils' growth.

A fourth means is that provided by pupils' behavior in what we might call unconstrained situations. Unfortunately, the student teacher must usually attempt some sort of evaluation long before he has an opportunity to see how successfully his pupils make use of what they have learned when they are "on their own." Nevertheless, this is most important. Evaluation is worthy of the name only when it is continuous. The student continues to reconstruct his teaching experience long after it is finished, in the light of new insights. If he can remain in touch with former pupils, so much the better.

The student teacher must keep in mind the obvious but nevertheless neglected truth that all evaluation is based on the desired outcomes, on the objectives which were formulated at the beginning of the work and on others which emerged as the work progressed. Thus evaluation is not in terms of comparisons of pupils' performance. The standard in mathematics is full understanding and perfect performance. No half-way ground is possible, nor would it be desirable if it were possible. Pupils must be given an opportunity to see clearly what is expected of them. Then they must be encouraged to work and study until such goals are reached. While acquiring new understandings they also gain skill in applying them. In addition, they should learn ways of checking

[84] See David Eugene Smith and William David Reeve, *The Teaching of Junior High School Mathematics*, Ginn and Company, 1927, Chapter XI.

their work. Evaluation under such circumstances becomes a cooperative affair between teacher and learner. The learners know what they have learned. They are ready and able to give evidence thereof. Further, they know their work is correct, for they have verified it themselves. Thus they are able to appraise their own progress. To compare one learner with another under such circumstances seems artificial, unwarranted.

It is to be expected that, in most situations, the student teacher must translate the evaluation of pupils' achievement into marks or grades. In doing so, he should make sure that marks are given in recognition of new learnings gained, of skills and understandings, rather than in recognition of attendance, "effort," "attitude," or "average" performance. He will encounter wide variations among teachers' techniques. Some teachers attempt to mark on the basis of the normal distribution curve, which means that the number of "A's" given must determine the number of "E's" (each about 7% of the total); likewise, the "B's" and "D's" (each about 20%). The remainder (about 46%) must be "C's" under this plan. Others employ the standard deviation of a given distribution of scores as the yardstick for assignment of marks. Whatever the practice is, he must make his adjustment to it; he must fit into the established scheme, at the same time doing his best to give marks sincerely and in terms of the intrinsic aims of mathematics. Later, when he is more experienced, he will reexamine the whole question of marking and attempt to find the most useful and valid bases.

In addition to outlining the student teacher's duties in the mathematics classroom, it is necessary to refer to his other duties in the school at large. In many schools the teacher of mathematics has a variety of responsibilities. He may be the sponsor of a chess club or some other hobby or service club, or he may serve as auditor of the student activities fund, or as debate coach, or as one of the administrative assistants. He may be a homeroom teacher. If his supervising teacher carries such responsibilities, the good student teacher does his best to share in them. If he has a hobby, he should not hesitate to let it be known. In many cases, his supervising or training teacher will encourage him to organize a group to promote such interests as stamp collecting, photography, making model airplanes, and the like. The additional time and effort invested in such work often bear direct relationship to the student's success, both while he is in his period of preparation and later.

In many secondary schools there is a movement to modify the compartmentalization of subject matter and organize the pupils under faculty chairmen or homeroom teachers who maintain contact with pupils throughout the school day. Such teachers are chosen for their broad experience and capacities. They coordinate plans for the groups under their care, calling in the mathematics teacher, the science teacher, and others, somewhat as music and physical education teachers are called in at special intervals to work with children in the elementary grades. Frequently the "specialist" is asked to contribute toward the development of some central theme, such as "Consumer-Producer Relationships." This kind of organization is sometimes designated as the *Core Curriculum*. In view of the changing character of our public secondary schools, the student teacher makes it his business to study such procedures with care, whenever he has the opportunity. By doing so, he is led to see how broad his preparation must be if he is to measure up to the demands of teaching, either as a core chairman or as a mathematics specialist, in a system organized along such lines.

After he has participated as observer, as assistant, and as teacher under supervision, the student teacher comes to the final phase of his experience, that of evaluating his own achievement. In reality, this is a continuous process, one which began when

he enrolled as a student teacher, and continues long past the time when he takes leave of his supervising teacher.

VII. How Evaluate the Teaching of Mathematics?

The ultimate test of any enterprise is found in the answer to the question, Did it work? The story is told that, one day, Ralph Waldo Emerson and his son were trying to get a balky calf into the barn. It seems that the calf was of good size and had resisted for some time the efforts of the philosopher and his son to get her through the barn door. Just as they were about to face defeat an Irish servant girl came along, saw their dilemma, put her finger in the calf's mouth, and led the animal into the barn. Emerson is reported to have written in his journal, referring to this incident, "I like people who can *do* things."

So it is with student teachers. Supervisors, pupils, administrators are drawn to people who can do things. Is there evidence that pupils have more confidence, more power in mathematics than before the student began teaching? Do they like mathematics even more than they did? Were new applications, relationships, possibilities for mathematics opened up to them? Is their thinking in all fields a bit more orderly, as a result of satisfactions they have found in mathematics? Have they become more independent, more self-reliant in the face of difficulties? Have they mastered some new concepts and processes by means of the student teacher's work? If the answer to each of these questions is "yes," then both the student teacher and his supervisors can look back on his experience with pride. They can look to his future teaching with assurance.

Means of judging the student teacher's work are found in the results of his testing. Not only the actual data from tests administered by the student teacher, but his manner of reacting to such results are valuable means of appraising his work. Did he use these results merely for marking purposes, or did he attempt to analyze the data for clues to guide him in reteaching? Toward the end of the school term, many supervisors administer standardized tests. Pupils' responses to these tests may then be compared with the performance of other groups, thus providing an index to the student teacher's teaching ability, especially when considered in connection with related psychological data.

In many institutions the supervisor and supervising or training teacher use check lists on which comparative ratings of student teachers may be marked concerning such traits as "appearance," and "command of subject matter." New Jersey State Teachers College[85] at Upper Montclair uses a card which provides an analysis of teaching ability as follows:

I. *Personal Qualities:*
Poise, bearing, grooming, voice, use of English, forcefulness, resourcefulness, enthusiasm, tact.
Suggestions:

II. *Teaching Skills:*
Developing and clarifying objectives; arousing interest; securing sustained, coordinated effort; leadership; adaptation of materials to needs of individual and group; use of equipment and accessories; evaluation of results.
Suggestions:

III. *Immediate Preparation:*
Systematic preparation of lesson plans; command of subject matter; provision for rich and suggestive supplementary materials; relating the business of the hour to the events and thoughts of the day in community, state, nation, and world.
Suggestions:

IV. *Management and Control:*
Management of room and equipment; attention to individual's needs; promptness and accuracy in routine; economy of time and effort; pupil citizenship.
Suggestions:

[85] State of New Jersey, Department of Public Instruction, *Principles and Practices in the Conduct of Student Teaching*, a syllabus prepared by the Department of Integration, New Jersey State Teachers College, Upper Montclair, New Jersey, p. 12, 1939. See also *Mathematics in General Education*, A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum of the Progressive Education Association. D. Appleton-Century Company, 1940. Chapter 13.

V. *Professional Relationships:*
Cooperation with high school and college; acceptance of criticism and suggestions; ability in self-analysis.
Suggestions:

VI. *Attainment of objectives:* growth of pupils; personal and professional growth of student teacher.
Suggestions:

Both the training teacher and the college supervisors evaluate the work of the student, using this analysis. They report their judgments to the Director of the Integration Department. (This corresponds somewhat to the Education Department in other schools for the preparation of teachers.) The Director of Integration thereupon assigns the student a letter grade for the course, based on these recommendations.

One device that student teachers consider helpful is for the training teacher to take notes on the work of the student teacher which are handed to him when the lesson is finished. One girl who had this experience said these notes were invaluable to her in her later work. This requires, of course, time and energy which some supervising teachers may be unwilling to spend in helping the novice. But it is a good plan, if well carried out.

Whatever the device used, such evaluations must consider the student teacher's progress in:

1. Learning to organize his own knowledge of mathematics for teaching purposes.
2. Discovering new interests in mathematics and becoming aware of ways to supplement his knowledge.
3. Becoming more sensitive to the problems of teaching mathematics.
4. Acquiring skill in teaching.
5. Finding ways of promoting his personal growth, such as: reading, association with scholarly people, membership in such bodies as the National Council of Teachers of Mathematics as well as in local mathematical and professional societies.

Very often the supervisor asks the student to write a resumé of his experience which includes answers to such questions as:

1. What were the greatest problems you faced during your terms as student teacher?
2. How and with what success did you attack these problems?
3. What features of the arrangements would you have changed if it had been in your power to do so?
4. In what part of the work did you find your greatest strength? Your greatest weakness?

Those responsible for evaluating the student teachers' work realize that their judgments are often crucial in the matter of the students' obtaining employment. Hence they weigh the evidence before them with great care, taking into consideration their obligations to society, to the student teachers themselves, as well as to the institutions they represent, bearing in mind that these institutions must vouch for their students later on.

Aside from the supervisor's evaluation, the student himself must appraise his work if it is to result in growth. In the privacy of his own thoughts he considers such questions as the following:

1. All things considered, what *should* I have done when I was engaged in student teaching?
2. What *could* I have done had I used my time and energies to the best advantage?
3. What *shall* I do, if and when another such opportunity comes my way, especially if I am given enough choice and time to carry out what I feel to be right and proper?

His answers to these questions will take into consideration his duties to pupils, parents, supervising or training teacher, supervisor, and—by no means least—to himself.

Thus the experience comes to an end. The student teacher has taken leave of the pupils and the supervising teacher. If he is well trained, he has paid a courtesy call on the school principal to thank him for the facilities which were made available for his student teaching. He has had his final conference with the supervisor. He has cleared his desk in the mathematics office or classroom, checked the supplies and materials he has used, and returned all the books borrowed from the library. If he is fortunate, he leaves knowing that he has made a friend in his supervising teacher. Now he looks forward to the next step, that of securing a permanent position.

◆ THE ART OF TEACHING ◆

An "Over-All" Method of Teaching Logarithms

By MAY L. WILT

West Virginia University, Morgantown, West Virginia

THE "piece-by-piece" method of teaching logarithms to high school students frequently leads to *manipulation*, without *understanding*. If this weakness is to be overcome, an "over-all" picture must be maintained throughout the whole teaching process. A brief discussion of a method in use in the West Virginia University High School follows:

INTRODUCTION TO LOGARITHMS

Basic to the study of logarithms is a thorough knowledge of exponents. It should include the:

1. Meaning of exponents,
2. Meaning of zero, negative, and fractional exponents,
3. Four fundamental operations effecting exponents—multiplications, division, raising to a power, and extracting roots,
4. Establishment of the laws of exponents resulting from these four operations,
5. Practice with mixed groups of exercises, including types such as these:

$(x^3)^4=$	$\sqrt{x^6}=$
$(x^6)^3=$	$x^6 \div x^2=$
$(x^{-3})^2=$	$\sqrt[3]{x^6}=$
$x^{1/2}\cdot x^{1/3}=$	$(10^4)^3=$
$x^{1/2}\div x^{1/3}=$	$x^3\cdot x^{2\ 1/2}$
$10^3\cdot 10^2=$	$10^{6.42}\div 10^{2.41}=$
$(10^{6.442})^3=$	$10^{1/2}\cdot 10^{2/3}=$
$10^{-6}\div 10^{-2}=$	$10^{1/2}\div 10^{2/3}$
$10^3\div 10^{1/2}=$	$10^{2.443}\cdot 10^{6.844}=$

$$\frac{(x^3)^4\cdot(x^2)^3\cdot(x^3)^3\cdot(x^{-2})^3}{(x^2)^4\cdot(x^{1/2})^6\cdot x^0}$$

$$\frac{(10^3)^2\cdot(10^2)^3\cdot(10^4)^3}{10^0\cdot(10^3)^5}$$

$$\frac{(10^{3.22})^3\cdot(10^{6.44})^2\cdot 10^4}{(10^{2.21})^3\cdot(10^{3.42})^2}$$

$$\sqrt{\frac{(10^3)^4\cdot(10^2)^2\cdot(10^3)^2}{(10^4)^3\cdot 10}}$$

$$\sqrt[3]{\frac{(10^{3.6642})^3\cdot(10^{6.2442})^2}{(10^{3.3311})^2}}$$

Many exercises are long and tedious, when solved by the common processes of elementary arithmetic. Verify this by working out *in detail* and by *keeping* a record of the time consumed in this exercise

$$\sqrt{\frac{(492)^3\cdot(492)^2\cdot(492)}{(4920)^2}}$$

Observe the number of opportunities for making errors.

COMPUTING THE SHORTER WAY

Recall that, by the use of like bases and the four laws of exponents, some startlingly short cuts may be made. The vital problem then is to learn how to *write numbers to the same base.*

1. Understand and expand the table of powers of 10, beginning with:

$$10^{-1}=0.1$$

$$10^0=1.$$

$$10^1=10.$$

2. Observe how the numbers in the original exercise may be placed in the table of tens.

$$10^0 = 1.$$
$$10^{0.+} = 4.92$$
$$10^1 = 10.$$
$$10^{1.+} = 49.2$$
$$10^2 = 100.$$
$$10^{2.+} = 492.$$
$$10^3 = 1{,}000.$$
$$10^{3.+} = 4{,}920.$$

3. Make a skeleton of the original exercise, verifying from the above table, as:

$$\sqrt{\frac{(10^{2\cdot}\text{——})^3\cdot(10^{1\cdot}\text{——})^2\cdot(10^{0\cdot}\text{——})}{(10^{3\cdot}\text{——})^2}}$$

4. Since the numbers lie between the multiples of ten, then, when they are written to the base of 10, their exponents must be between the powers above and below them. A number must be the power above it in the table, plus some fraction. This fraction, called the mantissa, is written in decimal form and can be found in a table of mantissas.
5. Using the table completes the process of writing the given numbers to the base of 10. The exercise now becomes:

$$\sqrt{\frac{(10^{2.6920})^3\cdot(10^{1.6920})^2\cdot(10^{0.6920})}{(10^{3.6920})^2}}$$

It is evident that:

a. the characteristic, or number to the left of the decimal point in the exponent, varies according to the size of the number or its position in the table of powers of 10.
b. the mantissa is the same when the order of the numerals is the same. If the order changes, the mantissa changes.

6. The operations in the original exercise are listed. After changing to the same base the replacement opertions are noted as:

raising to a power—multiplication of exponent by the power

multiplication—addition of exponents

division—subtraction of exponent of divisor from exponent of dividend.

extracting a root—dividing the exponent of the radicand by the index of the root.

7. Performing the operations, step by step, presents the following aspects:

$$\sqrt{\frac{10^{8.0760}\cdot 10^{3.3840}\cdot 10^{0.6920}}{10^{7.3840}}}$$

$$\sqrt{\frac{10^{12.1520}}{10^{7.3840}}}$$

$$\sqrt{10^{4.7680}}$$

$$10^{2.3840}$$

8. Know that the exponent of the power to which 10 must be raised to equal the given number is the logarithm of the number.
9. Understand that when an exercise, such as the one above, is reduced to a single expression—10 to some power—one more step becomes necessary. It must be changed back to a number that will be the answer to the exercise.
10. The number corresponding to a given logarithm is called the antilogarithm. This necessitates using a logarithm table in the reverse order from finding the logarithm. Looking up the mantissa inside the table and going out to find the number gives 242. From the table of powers of 10 a characteristic of two locates the number between 100 and 1,000. All numbers between these two locations consist of three places. Hence the location of the decimal point after the number thus: 242.

Additional Teaching Activities

The above steps serve to give the "over-all" picture of logarithms and to help the student understand what he is trying to do. However, a number of further details must be undertaken in completing the study of logarithms. They include:

1. The logarithm of fractions, written in decimal form,
2. Interpolating for logarithms and antilogarithms,
3. How to express the mantissa of any number in a form that will be positive,
4. The generalization of the laws for characteristics,
5. The meaning and use of cologarithms,
6. The writing and solution of exercises in the brief logarithmic form,
7. Historical material on John Napier and Henry Briggs,
8. A wide variety of practical exercises, including the compound interest formulas,
9. The exponential equation,
10. The slide rule—a concrete use of logarithms,
11. The reworking of the original exercise, in a complete manner, by logarithms, and a comparison of the time required by the arithmetical and by the logarithmic processes.

EDITORIALS

FROM HERE AND THERE

It has been suggested that a "From Here and There" page appear in The Mathematics Teacher, containing suggestions for the improvement of the National Council of Teachers of Mathematics, the official journal, and the professional standing of the members of the Council. What do our members and readers think of such an idea? Would such a page be of interest and possible value to you? Send your comments directly to the editor of The Mathematics Teacher.

Here are a few reactions to the idea of such a page from some of the State Representatives themselves:

1. I think the proposed question and suggestion page in The Mathematics Teacher is an excellent idea. The inclusion of a similar page in the *N.E.A. Journal* recently has been of benefit to many of us.

2. The "From Here and There" page might be profitable if the criticisms were constructive, but anyone can find fault.

3. Suggestions might be sent in that will be helpful if the writer's name is not published. A person will not really give his opinion if his name is going to appear with the suggestion.

4. If all the comments contain the name and address of the writer, it will give comment more weight and cause people to think twice before suggesting a "screw ball" idea.

5. I think the "Suggestion and Question" page in The Mathematics Teacher is a very good idea. Something which will enable us to become better acquainted with each other, to learn more about mutual problems and successful solutions thereof; this should be invaluable.

6. Your "From Here and There" is a good idea. It should be well worth two pages a month.

It is clear from 3 and 4 above that not all readers will agree, but there seems to be enough merit in the idea for us to ask for further comments.—W. D. R.

NOTES FROM STATE REPRESENTATIVES

Below are a few paragraphs taken from recent letters from State Representatives of The National Council of Teachers of Mathematics to the Chairman of State Representatives, Kenneth E. Brown, East Carolina Teachers College, Greenville, N. C.

1. I am just in the process of sending about a hundred letters to mathematics teachers here in. . . . Previous to that, in fact last month, I sent out about a hundred letters to teachers whom I hope to interest in becoming members of the Council. It is perfectly all right with me if I am not reimbursed for paper, postage, etc. My heart is in this work; I am glad to have this opportunity to work for the Council. My only regret is that results, in increased membership, seem to come so slowly. One would think that *every* mathematics teacher would consider it a duty and a privilege to support the only organization working solely for our interests as mathematics teachers. Moreover, The Mathematics Teacher is worth many times $2.00 a year.

2. Have you heard that one mathematics magazine is about to go out of existence? Just when mathematics has reached a favorable position in education, are we going to sit back and let mathematics drift into the dog house? We must all keep on working together.

3. I am writing letters to teachers in

larger towns of our state asking them to speak for THE MATHEMATICS TEACHER to the other teachers in their schools. If we are going to get any place with mathematics, we must work together.

4. I wrote an article for the State Bulletin which will appear soon, in which I asked the mathematics teachers to support the cause of mathematics by joining the National Council.

5. I have been encouraging the seniors in the mathematics department to become members of the Council and to read THE MATHEMATICS TEACHER regularly. I wonder if the Council would consider a student membership, possibly for a dollar, as the P.S.E.A. and N.E.A. do for the Future Teachers of America. I believe that would start prospective teachers on the right track if it is a possible scheme.

6. At present I am attending summer school and I may have an opportunity to publicize the work of the Council in a class on the teaching of arithmetic in the elementary grades.

7. We are starting an early campaign to get members to renew their membership in order to help relieve the clerical problem of dropping and adding members and prevent members from missing an issue of THE MATHEMATICS TEACHER.

The above comments show how hard some of our State Representatives are working to increase the membership of the Council and what real interest they have in the organization. We appreciate what they are doing. See their names and addresses on page 139.—W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
Midwood High School, Brooklyn 10, N. Y.

The American Mathematical Monthly

January 1947, Vol. 54, No. 1

1. Reeve, W. D., "Coordinating High School and College Mathematics," pp. 1–10.
2. Caton, W. B., "The Euler Column," pp. 11–16.
3. Mitchell, Ernest, "Conjugo-Conjugate Couples in Involution," pp. 16–23.
4. Fine, N. J., "The Jeep Problem," pp. 24–31.
5. Mathematical Notes, pp. 31–34.
   a. Grant, H. S., "A Linear Diophantine Equation"
   b. Walker, R. J., "An Artillery Problem"
6. Classroom Notes, pp. 35–38.
   a. Walsh, J. L., "A Rigorous Treatment of the First Maximum Problem in the Calculus"
   b. Vance, E. P., "Teaching Trigonometry"
   c. Reynolds, J. B., "Undetermined Coefficients in Integration"
7. Elementary Problems and Solutions, pp. 38–48.
8. Advanced Problems and Solutions, pp. 49–53.
9. Recent Publications, pp. 54–60.
10. Club and Allied Activities, pp. 61–64.
11. News and Notices, pp. 65–66.
12. The Mathematical Association of America—Official Reports and Communications, pp. 67–74.

School Science and Mathematics

January 1947, Vol. 47, No. 1

1. Graesser, R. F., "An Experimental Demonstration of e," pp. 9–13.
2. Nyberg, Joseph A., "Notes from a Mathematics Classroom" (Continued), pp. 76–80.
3. Coleman, Grace Marie, "The Signs of the Trigonometric Functions of Any Angle: a Mnemonic Device," pp. 81–82.
4. Problem Department.
5. Books and Pamphlets Received.

Scripta Mathematica

June 1946, Vol. 12, No. 2

1. Shaw, J. B., "Kaleidoscopic Rhythms," pp. 101–111.
2. Tarter, Harry, "The Successful Dinner Companion," (a poem), p. 112.
3. Kaplansky, Irving, and Riordan, John, "The Problème des Ménages," pp. 113–124.
4. Löwenheim, Leopold, "On Making Indirect Proofs Direct," pp. 125–139.
5. Vetter, Quido, "Czech Science during the War," pp. 141–146.
6. Book Reviews, pp. 147–152
7. Recreational Mathematics.
   a. Karapetoff, Vladamir, "Agha and Math," pp. 153–159.
   b. Grossman, Howard D., "Fun with Lattice Points," pp. 160–161.
   c. Gloden, A., "Multigrade Equation," pp. 161–162.
8. Miscellaneous Notes
   a. Wayne, Alan, "Inequalities and Inversions of Order," pp. 164–167.
   b. Guttman, Solomon, "Sums of Powers of Cyclic Numbers," pp. 167–169.
9. Curiosa
   122. Kraitchik, M., "A Common Illegal Operation," p. 111.
   123. Kraitchik, M., "Another Illegal Operation," p. 111.
   124. Piza, Pedro A., "An Interesting Identity," p. 146.
   125. Whitlock Jr., W. P., "Interesting Identities," p. 146.
   126. Gerard, R. W., "A Problem in Symmetry," p. 162.
   127. Guttman, Solomon, "Additional Links to Professor Golden's Chain," p. 163.
10. Quotations, p. 140
    12. A Forgotten Achievement.
    13. Veterans Problems—Old Style.
    14. What Is Greater than a Great Discovery?

Miscellaneous

1. Buswell, G. T., "Selected References on Elementary-School Instruction; Arithmetic" (continued). *Elementary School Journal*, 47: 161–163, November 1946.
2. Deans, E., "Meaningful Number Work," *Instructor*, 56: 28+, January 1947.
3. Gnaedinger, W. G., "Solids from Strings," *Educational Screen*, 25: 507–508, November 1946.
4. Lee, O. J., "Teaching of Science and Mathematics," *Popular Astronomy*, 54: 382–384, August 1946.
5. Rosenberg, R. R., "What Shall We Teach in Business Arithmetic?" *Business Education World*, 26: 220–222, December 1946.
6. Spitzer, H. F., "'Rithmetic Minus Hick'ry Stick," *National Parent-Teacher*, 41: 26–28, December 1946.
7. Van Engen, H., "Developing the Fraction Concept," *Nebraska Educational Journal*, 26: 340+, December 1946.
8. Wheat, H. G., "Changes and Trends in Arithmetic since 1910," *Elementary School Journal*, 47: 134–144, Nov. 1946.

State Representatives of the National Council of Teachers of Mathematics

Mr. J. Eli Allen
Phillips High School
Birmingham, Alabama

Dr. Robert J. Hannelly
Phoenix Junior College
West Thomas Road
Phoenix, Arizona

Miss Christine Poindexter
1905 Gaines Street
Little Rock, Arkansas

Mrs. Ruth G. Sumner
3000 Central Avenue
Alameda, California

Mr. H. W. Charlesworth
East High School
Denver, Colorado

Mrs. Marion Sharman Hancock
Box 180 Westville Ave. Ext.
Danbury, Connecticut

Mrs. Florence E. Loose
63 West Salisbury Drive
Edge Moor Terrace
Wilmington, Delaware

Mrs. Ethel Harris Grubbs
751 Fairmont Street
Washington, D. C.

Miss Veryl Schult
Wardman Park Hotel
Washington, D. C.

Dr. F. W. Kokomoor
University of Florida
Gainesville, Florida

Miss Bess Patton
1585 N. Decatur Rd. N. E.
Atlanta, Georgia

Miss Phillis Ellison
Route 2
Boise, Idaho

Dr. Miles Hartley
University High School
Urbana, Illinois

Dr. L. H. Whitcraft
Ball State Teachers College
Muncie, Indiana

Miss Dora Kearney
1105 West 23rd Street
Cedar Falls, Iowa

Professor Gilbert Ulmer
University of Kansas
Lawrence, Kansas

Miss Dawn Gilbert
1331 Clay Street
Bowling Green, Kentucky

Miss Jessie May Hoag
Box 181, S. L. I.
Lafayette, Louisiana

Miss Pauline Herring
128 Tolman Street
Cumberland Mills, Maine

Miss Agnes Herbert
806 East North Avenue
Baltimore, Maryland

Mr. Harold B. Garland
129 Houston Avenue
Milton, Massachusetts

Mr. Duncan A. S. Pirie
4628 Devonshire Road
Detroit 24, Michigan

Miss Edna M. Norskog
322 Fourth Avenue E.
Alexandria, Minnesota

Miss Lurine Stewart
Hinds Junior College
Raymond, Mississippi

Mr. G. H. Jamison
State Teachers College
Kirksville, Missouri

Mrs. Elsie May Wayne
739 Main Street
Hamilton, Montana

Miss Edith Ellis
1724 F. Street
Lincoln, Nebraska

Mr. John H. Fant
360 Moran Street
Reno, Nevada

Mr. H. Gray Funkhauser
Phillips Exeter Academy
Exeter, New Hampshire

Miss Mary C. Rogers
425 Baker Avenue
Westfield, New Jersey

Professor Harold D. Larsen
University of New Mexico
Albuquerque, New Mexico

Mr. H. C. Taylor
Ben. Franklin High School
Rochester, New York

Miss Henrietta L. Brudos
State Teachers College
Valley City, North Dakota

North Carolina
To be appointed

Mrs. Florence Brooks Miller
3295 Avalon Road
Shaker Heights
Cleveland, Ohio

Mr. J. V. Naugle
1541 Maple Avenue
Wyoming, Ohio

Miss Eunice Lewis
2203 East 14th Street
Tulsa, Oklahoma

Miss Sarah Ruby
1509 N. C. Tenth Avenue
Portland, Oregon

Miss Joy Mahachek
Head of Mathematics Dept.
State Teachers College
Indiana, Pennsylvania

Mr. M. L. Herman
Moses Brown School
Providence, Rhode Island

Miss Harriet B. Herbert
125 Broad Street
Sumter, South Carolina

Miss Josephine Wagner
Sioux Falls Schools
Sioux Falls, South Dakota

Mr. Robert L. Witt
Freed-Hardeman College
Henderson, Tennessee

Miss Mary Ruth Cook
North Texas State T. C.
T. C. Station Box 5206
Denton, Texas

Miss Le Nora Losee
1444 Kensington Avenue
Salt Lake City, Utah

Professor John G. Bowker
Middlebury College
Middlebury, Vermont

Miss Carrie B. Taliaferro
Box 102, State Teachers Coll.
Farmville, Virginia

Miss Helen Dunn
West Seattle High School
41st Ave. S.W. and W. Stevens St.
Seattle 6, Washington

Miss May L. Wilt
West Virginia University
Morgantown, West Virginia

Miss Theda Frances Howe
Riverside High School
Milwaukee, Wisconsin

Dr. C. R. Barr
University of Wyoming
Laramie, Wyoming

NEWS NOTES

Dr. Kenneth E. Brown, professor of mathematics at the East Carolina Teachers College at Greenville, N. C. spoke on "Modern Trends in Mathematics" at the 24th Annual Convention of the Northeastern District Teachers Association at Greenville on Nov. 15, 1946.

The officers elected at the meeting of the Iowa Association of Mathematics Teachers at Des Moines, Nov. 8, 1946, were the following:
President: Dowa E. Kearney, 1105 West 23rd Street, Cedar Falls, Iowa
Vice-President: Hesther Donthart, Newton, Iowa
Secretary-Treasurer: Dr. H. Vernon Price, University High School, University of Iowa, Iowa City, Iowa

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PRESIDENT'S MESSAGE*

What is your philosophy concerning your teaching job and education in general? Do you believe that each child in this day of mass education should be stimulated and challenged to the point of his maximum attainment? Do you believe that diplomas and degrees should mean that the recipient is truly educated or that he has merely put in a stipulated amount of time? Do you believe that we are spending too much time on the laggards at the expense of the willing and able pupils?

Many more questions such as these can and should be asked; however there is not room for more in this brief message. I merely mention the ones above to show that there is no time for complacency. At the present time the question of college entrance requirements is pertinent. Should they be altered, and if so, what form should the change take, and also who should take part in the proceedings? Obviously the question is too vast to be answered by hasty judgments. The correct answer will require the best efforts and deliberations of many experts in many fields.

Some of the reasons why we should meet together have been mentioned, but a few more are—for mutual stimulation and cooperation, fellowship, etc. It is the belief of the Executive Council that the Fall Meeting will do these things in greater or less measure and your presence will help to insure a satisfactory meeting for all concerned. Pass the word along and bring a friend with you. The fellowship luncheon is equally important for a renewal of old friendships and the acquiring of new ones.

The fee for membership in the Mathematics Section of the Colorado Education Association is **$1.00** per year which includes a subscription to the *Mathematics Bulletin*. You may also join the National Council of Teachers of Mathematics at this meeting for which the fee is **$2.00**, and includes a subscription to THE MATHEMATICS TEACHER.—Kenneth E. Gorsline.

* From the Bulletin of the Mathematics Section of the Eastern Division of the Colorado Education Association meeting at Denver, Colorado, Oct. 25, 1946.

BILL OF RIGHTS OF TEACHERS OF SECONDARY MATHEMATICS*

A Bill of Rights for the teacher of mathematics has a two-fold purpose. First it should declare his right to an *opportunity* for adequate preparation for the tasks which lie ahead. Second it should set forth his right to share fully in the *responsibility* associated with being a teacher of mathematics.

The nature of mathematics and its uses in the work-a-day world can be made of primary significance. This can be done by facing the facts with intelligence, courage, and patience. There is a desperate need for a meaningful understanding of relationships (both arithmetical and functional) as well as of the techniques of mathematical manipulation. The quantitative aspect of our current living offers both the opportunity and the responsibility for a fuller understanding through the mode of analysis commonly called mathematics.

Accordingly we believe a teacher of secondary mathematics has two sets of rights:

I. Those relating to opportunity:
   1. To expect colleges and universities to offer mathematics courses of more functional value than is being done.
   2. To study at first-hand the applications of mathematics to science, engineering, social science, and government under competent instructors.
   3. To have experience in business, in industry, and in government in order to become familiar with current practices in applications of mathematics.
   4. To expect that school boards will provide financial assistance for teachers to

* Prepared by a committee of Mathematics Teachers at the Mathematics Institute of Duke University July 3–12, 1945 consisting of the following persons:

W. W. Rankin, Chairman
Duke University, Mathematics

Walter H. Carnahan
Purdue University, Mathematics

J. Harold Goldthorpe
American Council on Education
Veryl Schult
Head of Secondary Mathematics
Washington City Schools, D. C.

Douglas E. Scates
Duke University, Education

William S. Schlauch
School of Commerce, N. Y. University

Walter J. Seeley
Duke University, Engineering

Ernest C. Thayer
Orleans Parish School, Mathematics

visit other schools and attend conferences and institutes.

5. To expect encouragement from school administrators for needful experimentation with recent developments in content materials and methods of instruction.
6. To expect a salary commensurate with his training and his responsibilities.
7. To have access to a mathematics laboratory with its library, illustrative devices, mathematical instruments, and teaching aids for classroom use.
8. To participate in curriculum building and adaptation of the curriculum to his students in mathematics and in the selection of textbooks.
9. To have satisfactory tenure provisions, and adequate certification standards.

II. Those relating to responsibility:

1. To acquire the knowledge and the skills needed in assisting students to understand and appreciate mathematics.
2. To become familiar with the vocational opportunities in his field and in related fields in order to guide students intelligently.
3. To see that students realize the broad objectives essential to good citizenship and satisfactory vocational performances.
4. To help establish and maintain high standards of excellence in teaching.
5. To encourage students to broaden their horizons by investigating quantitative relationships wherever they may be found.
6. To participate cooperatively in the best available in-service training.
7. To be familiar with the historical development of mathematics and its uses through the ages. (Such knowledge has both cultural and utilitarian values.)
8. To affiliate with such organizations as promote the study of mathematics on secondary level and stimulate his professional growth.

"Christine Poindexter, Little Rock, was recently elected Chairman of the Mathematics Section of the Arkansas Education Association for next year. Miss Irene Forrest was chosen as Vice-Chairman, and Miss Bess Johnson of North Little Rock as Secretary."

The establishment of a new department at Brown University to be known as the Graduate Department of the History of Mathematics was made public today by President Henry M. Wriston. The new department will be under the leadership of Dr. Otto Neugebauer, professor of the History of Mathematics. Associated with him as a key member of the staff will be Dr. A. J. Sachs, who was promoted to the rank of assistant professor of the History of Mathematics on Jan. 1.

Principal research objectives of the new department will be the study of ancient astronomy in its relations to mathematical disciplines and to the history of civilization. Entailed in this study is extensive description and interpretative work covering Babylonian, Egyptian, and Sanskrit material as well as data pertaining to Greek astronomy and astrology and ancient chronology in general.

From the standpoint of instruction, a course in Oriental History, open to both graduate and undergraduate students, to be taught by Professor Sachs, is planned for the academic year 1947–48. It is also expected that a seminar on selected topics in ancient astronomy and mathematics will be offered in cooperation with the departments of mathematics, astronomy and classics.

Commenting on the establishment of the new department, Dr. Samuel T. Arnold, dean of the University, stated, "Professor Neugebauer is one of a small group of scholars in the fields of ancient history, mathematics and astronomy who works from the original cuneiform and hieroglyphic source materials. A member of the Brown department of mathematics since 1939, Professor Neugebauer is recognized as a top-ranking authority in his field."

Dr. Neugebauer attended the universities of Graz, Munich and Göttingen, receiving his Ph.D. from the latter in 1926. He is the first to have interpreted many Babylonian cuneiform tablets, some of them dating from the second millenuium before Christ. He has discovered that algebra used by the Greeks was known to the Babylonians 2,000 years earlier and has thrown much light on the Babylonian system of astronomy.

From its inception in 1931, until 1939, Dr. Neugebauer was editor-in-chief of "Zentralblatt für Mathematik," international abstract journal for mathematics published in Germany. He resigned because of political interference with the editorial management. Prof. Neugebauer was a faculty member of the University of Göttingen from 1927 to 1933 when he removed to the University of Copenhagen. Member of numerous educational and scientific organizations, Prof. Neugebauer was last spring elected a fellow of the American Academy of Arts and Sciences.

Professor Sachs, who will collaborate in the development of the new department, has been associated with Brown since 1941 when he joined the mathematics department through a special fellowship provided by the Rockefeller Foundation. During the period 1943 to 1946 he has been a research associate in the same department.

From 1939, when he received his Ph.D. from the Johns Hopkins University, until 1941, Professor Sachs was a research assistant in the Oriental Institute of the University of Chicago. He is a native of Baltimore and did undergraduate work at Baltimore City College.

Authorities at Yale, Johns Hopkins and the University of Chicago, as well as Brown, rank Professor Sachs as foremost among the younger generation of experts concerned with the history of ancient civilization. He is known as an Assyriologist who specializes in the historical, linguistic, and paleographical aspects of the field. He is a member of the American Oriental Society and the Linguistic Society of America, and is co-editor of the *Journal of Cuneiform Studies.*

1947 Summer Session at Teachers College, Columbia University in the Teaching of Mathematics

Teachers College, Columbia University, will offer the following courses in the teaching of mathematics in the summer session of 1947, which begins on July 7, and ends on August 15.

By Professor John R. Clark: Teaching algebra in secondary schools; teaching arithmetic in the elementary school.

By Professor Howard F. Fehr: Professionalized subject matter in junior high school mathematics; professionalized subject matter in advanced secondary school mathematics.

By Dr. Nathan Lazar: History of mathematics; logic for teachers of mathematics.

By Mr. Gordon R. Mirick: Elementary mechanics (statics); observation and participation in the teaching of geometry.

By Professor William D. Reeve: Teaching and supervision of mathematics—junior high school; teaching and supervision of mathematics—senior high school.

By Professor William S. Schlauch: Business mathematics.

By Professor Carl N. Shuster: Teaching geometry in secondary schools; field work in mathematics.

There will be held during the summer session, on consecutive Thursdays beginning on July 10, five special lectures and discussions in which all of the instructors above and other persons from outside will participate for the purpose of bringing before the students vital questions relating to the reorganization and teaching of mathematics in the post-war world. There will be an opportunity for discussion in which all of the students will be invited to take part. These conferences have come to be a common meeting place of all students, instructors, and guests, and thus serve both professional and social ends. Registration for these special lectures and discussions is not necessary and all those who are interested in the improvement of mathematical education are invited to attend.